

TSG 16 – Agenda

Updated version on July 13th 2021

TSG 16: Reasoning, argumentation and proof in mathematics education

Class: B

28 presentations: 10 long oral, 18 short oral

Four sessions – the three Class B sessions and an extra Class A session: 19:30-21:00 Beijing time, July 14th

Session 1 - 19:30-21:00 Beijing time, July 13th 2021

Chair : Kotaro Komatsu (online)

19:30-19:45 Presentation of the four sessions

1. Time: 19:45–20:00

Title of the Paper: WRITING A PROOF TEXT AT THE UNIVERSITY LEVEL: THE ROLE OF KNOWING WHAT A PROOF IS

Author: Nadia AZROU

Institution: University Yahia Fares, Algeria

Short abstract of the paper:

This paper examines the role of the knowledge of what proof is, particularly, regarding its relationship with proof text writing. Third year university students had to build up the proofs for three proving-problems. I have investigated the knowledge of what proof is of both students whose proof tests were of poor quality, and students with proof texts of good quality. The concept of proof was framed according to Vergnaud's theory of conceptual fields; Brousseau's didactic contract has also been taken into account. In this paper, I want to make explicit and precise the relationship between meta-knowledge of proof (revealed by the interviews as dependent on the didactic contract when it is false) and those salient aspects of proof texts, which are related to meta-knowledge of proof.

2. Time: 20:00-20:15

Title of the Paper: FORMALISATION OF PROOF. A TOOL FOR RESEARCHER

Author: Faiza CHELLOUGUI

Institution: Faculty of Sciences of Bizerte - Univ. of Carthage, Bizerte, Tunisia

Short abstract of the paper:

In this paper, we propose to illustrate the pertinence and potentialities of the natural deduction system of Copi (1954) for a didactical study of the mathematical proofs activity by emphasizing the originality of the analysis methodology which we developed leaning on this system (Chellougui, 2018). We will present in the first part the main results of the epistemological survey on the emergence of logical formalism, which contributed to our didactic questioning.

We will show that the formalization of mathematical statements in the language of predicate calculus reveals implicit quantifications and makes it possible to study the complexity of statements structure. In a second part, we will present the methodology that we have developed based on the Copi's natural deduction to conduct the a priori and a posteriori analyzes of the mathematical proofs. We will illustrate this methodology by presenting an experimental study conducted with first-year mathematics undergraduate students who have take an academic course of Copi's natural deduction system. We will present the experimental system and the main results obtained.

3. Time: 20:15-20:20

Title of the Paper: STUDENT INTERPRETATION OF DIAGRAM IN HYPERBOLIC GEOMETRY: CHANGES IN THE ONTOLOGY OF GEOMETRIC MODELS

Author: Younggon BAE

Institution: University of Texas Rio Grande Valley, Korea

Short abstract of the paper:

in this paper, I present an empirical study on students uses of Dynamic Geometry Environments (DGEs) and their engagement in mathematical reasoning and axiomatic reasoning while they enacted a task sequence in an axiomatic geometry course. Students used DGEs to communicate their mathematical ideas and to examine mathematical statements describing properties of geometric objects in Klein model of hyperbolic geometry. The findings of the analysis provided theoretical implications to better understand the nature of student engagement in advanced mathematical reasoning in such technology-rich environments. In particular, this presentation highlights students interpretations of parallel lines and congruent line segments in Klein model and discuss epistemological aspect of student reasoning regarding changes in the ontology of non-Euclidean geometric models in DGEs.

4. Time: 20:20–20:25

Title of the Paper: A COMPARATIVE STUDY OF EXAMPLE USE IN THE PROVING-RELATED ACTIVITIES OF KOREAN AND AMERICAN STUDENTS

Authors: **GwiSoo NA**, Eric KNUTH

Institutions: Cheongju National University of Education, Cheongju-si, Chungcheongbuk-do, Korea; University of Texas at Austin, United States of America

Short abstract of the paper:

Examples play a critical role in mathematical practice as the time spent thinking about and analyzing particular examples can provide not only a deeper understanding of a conjecture but also insight into the development of a proof. Yet, little research has focused on the nature of secondary school mathematics students thinking about and use of examples in creating, understanding, and proving mathematical conjectures. In this paper, we report on two studies that examined secondary school mathematics students thinking about and use of examples as they engaged in proving-related activities. In particular, results are presented from two studies in which Korean students (Study 1) and American (United States) students (Study 2) engaged in proving-related activities with the same set of conjectures. The results highlight both similarities

and differences in the example use students exhibited, and implications of these results are briefly discussed.

5. Time: 20:25-20:30

Title of the Paper: WHEN IS AN ARGUMENT AN ARGUMENT? AREA-SPECIFIC ASPECTS OF ARGUMENT-REZEPTION

Authors: **Michael MEYER**, Christoph KOERNER, Julia REY

Institution: University of Cologne, Cologne, Germany

Short abstract of the paper (20 lines maximum):

The importance of proof is undisputed in mathematics and its didactical discussion. Nevertheless, when the question arises What counts as a proof?, opinions divide. In the present study, an expert-novice comparison was used to examine how students of different 8th grade classes and students at the beginning of university level asses given arguments and to recognize their perspectives on arguments.

6. Time: 20:30-20:35

Title of the Paper: ARTICULATION OF ARGUMENTATION AND MATHEMATICAL MODELLING IN THE MATH CLASSROOM

Authors: **Horacio Cristian SOLAR**, Manuel GOIZUETA, Maria ARAVENA-DIAZ, Andres Ivan ORTIZ JIMENEZ

Institutions: Pontificia Universidad Catolica de Chile; Universidad Católica del Maule; Universidad Católica de la Santísima Concepción

Short abstract of the paper:

Although many comprehensive research publications deal with modelling and argumentation, these lines have been developed independently. Within the context of a study proposing to characterise the students learning by fostering an articulate promotion of the argumentation and modelling competencies in the mathematics classroom this contribution describes a classroom data analysis strategy, based on the theoretical fundaments of the study. The designed analysis strategy consists of different coding phases aimed at obtaining a complex image of the students mathematical activity and learning opportunities. This was achieved by the observation and analysis of 3 to 5 classes of 11 teachers who participated in a multiple cases study. Classes were based on modelling tasks and orchestrated through argumentation.

7. Time: 20:35-20:40

Title of the Paper: FOSTERING THIRD GRADERS FRACTION CONCEPTIONS THROUGH ARGUMENTATION AND TECHNOLOGY

Author: Ho-Chieh LIN

Institution: The Ohio State University- STEM Education, Columbus, United States of America

Short abstract of the paper:

This study investigated how argumentation and technology support third-grade students fraction conceptions as they engage in equal-sharing story problems with the use of tablets and

paper and pencil. Findings include: first, 3rd graders would regard any fractional part as a half. Children may also confuse with the meaning of halves. Second, the analogy of using integer magnitudes seems intuitive and helpful for children to reason fraction magnitudes. Third, symbols support childrens' reasoning and communication. Children may not articulate in expressing their thinking, but pictorial representations they draw on either tablet or paper helps facilitate explanation and interpretation, leading to their (re) construction and appropriation of symbols Fourth, argumentation appears conducive to participants conceptual understanding as children build on each others ideas, modify their thinking, and try to propose their explanation to convince themselves and others. Fifth, a teacher plays a crucial role as they should attend to children's thinking, specify focused concepts, inquire explanations, and facilitate argumentation.

8. Time: 20:40 – 21:00

Collective discussion on the five short oral presentations

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Session 2 – 19:30-21:00 Beijing time, July 14th 2021

Chair: Nadia AZROU (online)

1. Time: 19:30–19:45

Title of the Paper: CHANGES IN THE ARGUMENTATION CHARACTERISTICS OF MATHEMATICALLY GIFTED STUDENTS - A LONGITUDINAL STUDY

Authors: **Simone JABLONSKI**, Matthias Dieter LUDWIG

Institution: Goethe University, Frankfurt, Germany

Short abstract of the paper:

The changes and development of argumentative skills in the context of a deeper mathematical understanding are important aspects of mathematical teaching and learning, especially in the support of gifted children. This longitudinal study documents the mathematical argumentation products of potentially mathematically gifted primary school children over a 12-month period. Within this period, 14 children worked on arithmetical reasoning tasks in individual interview settings. For each child, the study involves three interviews. This spans the childrens age from nine to ten years. The changes in the hereby formulated argumentation products are analyzed on the levels (1) Toulmins argument structure, (2) content of the argument, (3) independency of the argument, and (4) validity of the argument. The paper provides first insights into the hypothetical consistency and variability of the characteristics of the children's arguments.

2. Time: 19:45–20:00

Title of the Paper: AN INQUIRING-GAME FOR DISCOVERING AND PROVING A GEOMETRIC THEOREM

Author: Carlotta SOLDANO

Institution: University of Torino, Torino, Italy

Short abstract of the paper:

This paper focuses on the exploration and argumentation processes activated by students involved in a didactic activity, called inquiring-game activity, composed of a game based on a geometric theorem and of a worksheet containing guiding questions. The design of this activity is inspired to the Logic of Inquiry and implemented inside GeoGebra. The analysis of the productions of two videotaped students is meant to show the deep intertwinement between regressive and progressive forms of reasoning which characterized the conjecturing phase. Students conjectures evolution is described by different types of controls: perceptive, theoretical and instrumental. Our hypothesis is that the design of inquiring-game activities can bridge the gap between the conjecture generation and the proof production, paving the way for the appearance of cognitive unity.

3. Time: 20:00–20:05

Title of the Paper: THE FUNCTION OF DEFINITION IN JAPANESE TEXTBOOKS

Author: Shogo MURATA

Institution: University of Tsukuba, Tsukuba, Japan

Short abstract of the paper:

This study aims to discuss the function of definition in Japanese secondary-school mathematics textbooks. Using a two-dimensional framework based on the work of Morgan (2006), the function of definition itself and the position of the user with respect to the definition in the textbooks are investigated. The results show that (i) the textbooks refer not only to the distinguishing function of definition but also to the logical-argument function of definition and (ii) students are positioned as constructors of definitions in one textbook and as receivers of definitions in another. This study suggests that it is important to design situations in which students can engage in defining activities.

4. Time: 20:05–20:10

Title of the Paper: INVESTIGATING THE DIFFERENCE BETWEEN GENERIC PROOFS AND PURELY EMPIRICAL VERIFICATIONS

Author: Leander KEMPEN

Institution: University of Paderborn, Paderborn, Germany

Short abstract of the paper:

The teaching of generic proofs has produced interest across the globe. Besides the advantages generic proofs offer for learners, the use of concrete examples in the context of general verifications might foster students misconceptions about the epistemological value of purely empirical verifications. The study presented in this paper is about the results of a proof questionnaire exploring pre-service teachers ability to distinguish different usages of examples in the context of mathematical proof. The study shows that a specifically designed course making use of generic proofs may help students to issue correct judgements on the epistemological value of concrete examples in the context of proving

5. Time: 20:10 –20:15

Title of the Paper: PROOF AND REASONING IN HIGH-STAKES TESTING SYSTEMS: THE SENIOR SECONDARY MATHEMATICS CURRICULA IN HONG KONG AND INTERNATIONAL BACCALAUREATE DIPLOMA PROGRAMME

Author: Chun-Yeung LEE

Institution: University of Oxford, United Kingdom

Short abstract of the paper:

Proof can verify the truth of a mathematical statement, promote sense making and understanding in mathematics through explanation, and promote character building and positive morality of mathematics learning through exploration. Many mathematics education researchers suggest that proof and reasoning should play central role in school mathematics at different levels and across different content areas. To investigate the roles of proof played in a senior secondary mathematics curriculum, I suggest to analyse its curriculum documents, namely, curriculum guides and exam papers. In this study, I compared the curriculum documents of Hong Kong and International Baccalaureate Diploma Programme, and investigated how different the roles of proof were expected in the two systems. Based upon my analysis, I discussed three conditions required to promote or centralize proof in an examination-driven curriculum.

6. Time: 20:15–20:20

Title of the Paper: MATHEMATICS CLASSROOM ARGUMENTATION: AN INTERACTIONAL PERSPECTIVE

Author: Markos DALLAS

Institution: University of Agder, Kristiansand, Norway

Short abstract of the paper:

A review of the related literature indicates major notions that could characterize an interactional perspective on argumentation in the mathematics classroom: those of argumentative discourse; behaviour, norm and (mathematical) practice; and participation and participant role. In order to conceptually organize these ideas, a model called the Mathematics Classroom Interactional Model (MCIM) is developed which also guides and supports the methodology of my PhD study. In this paper, an attempt is made to operationalize the MCIM model by formulating the research questions of the study and discussing an initial stage of the methods of analysis.

7. Time: 20:20–20:25

Title of the Paper: UNDERSTANDING THE GENERALITY OF MATHEMATICAL STATEMENTS AND THE ROLE PROOFS PLAY

Author: Milena DAMRAU

Institution: Bielefeld University, Germany

Short abstract of the paper:

In this paper, an experiment to test the understanding of the generality of mathematical statements is presented. It investigates, if people who are provided with a correct deductive (symbolic or generic) proof for a statement are more or less aware that there cannot be any

counterexamples to the statement, than people who only get empirical arguments or no arguments at all. The experiment will be conducted during a lecture in the first semester of pre-service primary and middle school teachers.

8. Time: 20:25–20:30

Title of the Paper: IS THERE ANY DIFFERENCE IN STUDENTS' DESCRIPTIONS DUE TO DIRECTION DIFFERENCES IN A DEDUCTIVE REASONING TASK?

Authors: **Yoshiki SHIBATA**, Tadashi MISONO

Institution: Shimane University, Japan

Short abstract of the paper:

The purpose of this study is to clarify differences of understandability and suitability of representations as proof in descriptions written by 9th grade students in Japan due to differences of direction of a deductive reasoning task; 'to prove' and 'to explain'. As a result, students' descriptions varied because each student valued various points. In addition, there was no significant difference in the two directions regardless of the difference in task directions. Moreover, we found that evaluators in this study valued logical descriptions regardless of their representation styles.

9. Time 20:30-20:50

Collective discussion of the six short oral presentations

10. Time 20:50-21:00

Some issues raised in the two first sessions

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Session 3 - 21:30-23:00 Beijing time, July 16th 2021

Chair: Samuele ANTONINI (online)

1. Time: 21:30–21:45

Title of the Paper: COMPUTER-ASSISTED PROVING IN THE CLASSROOM

Authors: **Xiaoheng (Kitty) YAN**, Gila HANNA

Institutions: Simon Fraser University, Burnaby, Canada; OISE, University of Toronto, Canada

Short abstract of the paper:

This paper argues that there is a need to develop new approaches to the teaching of proof that capitalize not only on newly available technology but also on modern theories of teaching and learning. It contributes to filling this need by opening a discussion on proof assistants that allow one to do mathematics with the aid of a computer, in particular to construct proofs and to check their correctness. The paper argues that proof assistants could play an important role in fostering students appreciation and understanding of proof and of mathematics as a whole. It

also underscores the need to develop explicit pedagogic strategies tailored to assist teachers in deploying such computer-based tools.

2. Time: 21:45–22:00

Title of the Paper: AN APPLICATION OF HABERMAS' THEORY OF VALIDITY CLAIMS FOR CLASSROOM-BASED ARGUMENTATION

Authors: **Yuling ZHUANG**, Anna Marie CONNER

Institution: University of Georgia, Athens, United States of America

Short abstract of the paper:

Little consensus existed about general acceptance criteria for classroom-based mathematical argumentation so far. Building on Habermas's (1984) theory of validity claims, this study developed an analytical framework to capture "validation" of an argumentative discourse with respect to three forms of validity claims: propositional truth, normative rightness, and sincerity. The framework may help the field to identify the fine-grained conditions for valid argumentation and support teachers to be aware of different forms of valid argumentation when attempting to support classroom-based argumentative practices.

3. Time: 22:00–22:15

Title of the Paper: CHARACTERIZING MATHEMATICS TEACHERS PROOF-SPECIFIC KNOWLEDGE, DISPOSITIONS AND CLASSROOM PRACTICES

Author: **Orly BUCHBINDER**, Sharon MC CRONE

Institution: University of New Hampshire, Durham, United States of America

Short abstract of the paper:

In many countries, there has been an increased emphasis on engaging students in reasoning and proof across grade levels and mathematical topics. To realize this kind of teaching in classrooms, teachers themselves must have Mathematical Knowledge for Teaching Proof (MKT-P), knowledge of proof-related practices and productive dispositions towards proof. Grounding our work in existing research, we propose a theoretical framework for MKT-P and illustrate how this framework inspired the development of two complementary instruments: an MKT-P questionnaire and Lesson Enactment rubric. We also describe a Dispositions towards Proof survey, which captures the aspects of teachers proof-related beliefs and confidence (or the lack of thereof) in teaching proof. Taken together, the three instruments allow for characterizing teachers professional resources specific to proof, which, in turn, can help to explain and advance aspects of teachers classroom enactment of proof.

4. Time: 22:15–22:20

Title of the Paper: A COMPARATIVE STUDY OF GEOMETRIC PROOF OPPORTUNITIES IN TAIWAN AND MAINLAND MIDDLE SCHOOL TEXTBOOKS

Authors: **Lei HAO**, P-Jen LIN

Institution: Mathematics and Science Education, NTHU, China

Short abstract of the paper (20 lines maximum)

The learning and teaching of proof has always been a research topic of Mathematics education, geometry is linked with the proof, and textbooks is an effective factor for learners to study proof, so the purpose of this study is to explore the geometric proof opportunities through development and exercise activities in Taiwan and mainland Middle school mathematics textbooks to provide resources and references for textbook writers in the future. Through content analysis, the analytic framework of this study is based on Otten et al.(2014) geometric proof textbooks analysis framework. This analysis of two Middle school textbooks characterizes the types of the statements, questions, and the process of the justification given in the exposition, examples and the exercises. The results show that the questions in the different mathematics textbooks have their own characteristics, and the corresponding suggestions for improvement are put forward.

5. Time: 22:20–22:25

Title of the Paper: USING WRITING AND DISCUSSIONS TO SUPPORT MATHEMATICAL ARGUMENTS IN EARLY ALGEBRA

Authors: **Salvador HUITZILOPOCHTLI**, Daniel LOPEZ-ADAME, Judit MOSCHKOVICH

Institution: University of California- Santa Cruz, Santa Cruz, United States of America

Short abstract of the paper:

This study examined how writing supported students mathematical arguments in early algebra. Following a FALS lesson (MARS, 2019) on consecutive sums using writing and revision with middle school students, we analyzed student arguments, claims, and justifications. Analysis showed that one group of students consistently used a process view of algebra expressions while the other group used an object view. The findings suggest that revising and discussing written arguments can support students in moving from a process to an object view.

6. Time: 22:25–22:30

Title of the Paper: JUSTIFICATIONS IN EXPOSITION IN ALGEBRA IN SCHOOL MATHEMATICS TEXTBOOKS IN HONG KONG

Author: Kwong Cheong WONG

Institution: The Hong Kong Polytechnic University, Honk Kong

Short abstract of the paper:

Justifications in the exposition sections in algebra of a popular senior secondary school mathematics textbook series in Hong Kong are examined. More non-proofs than proofs and far more statements being unjustified than left to students to justify are found. These results suggest that proof is not emphasized in school mathematics in Hong Kong.

7. Time: 22:30–22:35

Title of the Paper: DIFFERENT TYPES OF REASONING IN GEOMETRY IN BRAZILIAN HIGH SCHOOL MATHEMATICS TEXTBOOKS

Author: Lucas CARATO MAZZI

Institution: Unesp, Rio Claro, Brazil

Short abstract of the paper:

The purpose of this paper is to discuss different types of reasoning used in Geometry chapters of seven collections of high school textbooks from Brazil. Regarding the teaching of mathematics, different types of reasoning are relevant for a complete overview of how mathematics is developed. In this paper, we intend to discuss four types of reasoning deductive, inductive, abductive and reasoning by analogy - present in the Geometry chapters of 21 High School textbooks approved by the 2018 National Textbook Program (PNLD). This research has a qualitative approach, since the intention is to elaborate a deep interpretation about how textbooks present those reasoning throughout their Geometry chapters. The results point out that all books have both deductive and inductive reasoning, but do not connect them. The abductive reasoning was found in only one collection of books and the reasoning by analogy was found in some tasks.

10. Time: 22:35– 23:00

Collective discussion on the four short papers and on general issues.

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Session 4 – 14:30-16:30 Beijing Time, July 17th 2021

Chairs: Viviane Durand-Guerrier (online) & Zhou Chao (Shanghai)

1. Time: 14:30–14:45

Title of the Paper: COGNITIVE CHARACTERISTICS GENERATING INCOMPLETE PROOF: ANALYZING THE SOLVING PROCESS OF A GEOMETRICAL PROBLEM BY JAPANESE NINTH GRADERS

Author: Tomohiko MAKINO

Institution: Utsunomiya University, Japan

Short abstract of the paper:

Why is incomplete proof generated? In this paper, I investigate the generating process of incomplete proof. I approach the cognitive characteristics of generating incomplete proof by focusing on modes of expansion of the discourse and coordination activities in solving a geometrical problem by six pairs of Japanese ninth graders. By investigating the generating process of incomplete proof, I recorded the students' perspectives on accumulation and substitution, then invented codes for three statuses of coordination to grasp the aspects of coordination activities, and analyzed the process of problem-solving by the students. The results of this research demonstrate that cognitive characteristics that generate incomplete proof as dysfunction of substitution and pending coordination.

2. Time: 14:45–15:00

Title of the Paper: CAUGHT IN-BETWEEN TENSIONS IN TEACHING PROOF AND PROVING

Authors: Sikunder ALI, **Trond Stoelen GUSTAVSEN**, Sigurd Johannes HALS, Andrea HOFMANN, Silje TRAI

Institution: University of South-Eastern Norway, Drammen, Norway

Short abstract of the paper:

Proof and proving are central for the learning of mathematics at all levels of schooling. How mathematics teachers put proof and proving into practice requires attention to actual practice and views of mathematics. Inspired by activity theory we present results from two interview-based studies in the Norwegian context in forms of tensions that a mathematics teacher faces in handling varied and complex demands associated with the action of teaching proof and proving in mathematics. We conclude the paper by recommending research on tensions and dilemmas that a mathematics teacher faces in handling varied and complex demands associated with the action of teaching proof and proving in mathematics.

3. Time: 15:00–15:15

Title of the Paper: POSING NEW RESEARCHABLE QUESTIONS AS A DYNAMIC PROCESS: THE CASE OF RESEARCH ON STUDENTS' JUSTIFICATION SCHEMES

Authors: **Andreas STYLIANIDES**, Gabriel STYLIANIDES

Institution: University of Cambridge, Cambridge, Great Britain

Short abstract of the paper:

In this theoretical paper we argue that posing new researchable questions in educational research is a dynamic process that reflects the fields growing understanding of the web of potentially influential factors surrounding the examination of a particular phenomenon of interest. We illustrate this thesis by drawing mainly on mathematics education research related to students' justification schemes that has evolved rapidly during the past few decades. We also consider briefly another example in the area of teachers knowledge. Finally, we discuss implications of the thesis for findings of past studies related to the phenomenon of interest, for the design of new studies of the same phenomenon, and for the evolving nature of research knowledge about the phenomenon.

4. Time : 15:15-15:20

Move from online presentation to physical presentation

5. Time: 15:20–15:25

Title of the Paper: ANALYSIS OF ANALOGICAL REASONING EXERCISES IN PRIMARY SCHOOL MATHEMATICS TEXTBOOKS: TAKING GEOMETRY FIELD AS AN EXAMPLE

Authors: **Yaoyao DONG**, Jian LIU

Institution: Beijing normal university, Beijing, China

Short abstract of the paper:

Analogical reasoning plays an important role in mathematics learning. The research on analogical reasoning in cognitive psychology can provide psychological basis for the analogical reasoning teaching. Based on the process of analogical reasoning as an analytical framework, this paper analyzed the analogical reasoning exercises about geometry in the two main primary school mathematics textbooks in mainland China from four aspects of retrieval, mapping, inference and evaluation. Results shows that: Most exercises restrict the source analogs; All exercises involve the mapping and related inferences of the relationship of analogies; There is no

clear requirement for students to evaluate and test the inferences. Based on it, suggestions are provided for the improvement of analogical reasoning teaching.

6. Time: 15:25–15:30

Title of the Paper: REGIONAL AND GENDER DIFFERENCES IN CHINESE 8TH GRADE STUDENTS' MATHEMATICAL REASONING COMPETENCY

Authors: **Xin ZHENG**, Jing CHENG

Institution: East China normal university, shanghai, China

Short abstract of the paper:

The purpose of this study is to assess the overall level of Chinese eighth graders' mathematical reasoning competency and whether there are regional and gender differences. A total of 1464 students participated in the study and the results showed that in the performance of plausible reasoning competency, most students can solve moderately level problems. But their deductive reasoning competency varied greatly. Students in eastern and central regions had a higher level in reasoning than those in western regions. Male students reasoning competency was significantly higher than females.

7. Time: 15:30-15:35

Title of the Paper: A STUDY OF THE TEACHING PROCESS OF MATHEMATICAL CONCEPT ARGUMENTATION BASED ON TAP-- TAKING FUNCTION CONCEPT TEACHING BETWEEN EXPERT TEACHER AND NOVICE TEACHER IN CHINA AS A CASE

Authors: **Yi ZHANG**, Xiaopeng WU

Institution: East China Norm University, China

Short abstract of the paper:

Reasoning and argumentation ability is key ability of mathematical learning. Especially in the learning of important concepts, it is more important to strengthen the system argument process of concepts. Toulmin's Argument Pattern (TAP) can more specific and in-depth analyze the argument process of concept teaching through making a structured analysis, and grasp the teacher's classroom teaching argument level. This study selects 5 groups of expert and novice teachers' high school function concept teaching in China as the research object. Based on TAP, the two types of teachers' classroom teaching argument process are analyzed statistically from the two dimensions of the number of elements of the concept and the level of the argument level of the concept. It is concluded that novice teachers have a wide gap with expert teachers both in terms of the richness of the elements of concept argumentation and in the depth of the argument. In order to get some inspiration to providing reference for improving the teaching level of concept teaching.

9. Time 15:35 – 15:45

Collective discussion on the three short papers

11. Time: 15h45 – 16h30

Discussion on Future research agenda and possible collaborations