ICME-14 TSG-9

Teaching and learning of geometry at secondary level

Chair: Keith Jones (University of Southampton, UK)

Co–chair: Matthias Ludwig (Goethe University Frankfurt, Germany)

Team members: Liping Ding (Norwegian University of Science and Technology, Norway), Joris Mithalal (University of Lyon, France), Yiling Yao (Hangzhou Normal University, China)

IPC Liaison Person: Maria Alessandra Mariotti (Italy)

TSG9 brings together ICME-14 participants to share research results, research projects, new developments, and updates on ongoing projects concerning geometry at the secondary school level. Presentations and discussions are likely to focus on the following themes:

- Connections between geometry education and mathematical practices and processes such as argumentation and proof, visualization, figuration, and instrumentation;
- Teacher preparation and teacher knowledge for geometry at the secondary school level;
- Developments in teaching, including geometrical modeling and out-of-school problem solving;
- Curricular issues in school geometry, including reform initiatives in school geometry, and new forms and applications of geometry;

TSG-9 has three sessions and, as is standard practice at an ICME, TSG contributors attend, and can contribute to, *every session of the TSG* by presenting (at the scheduled time) or by using the 'chat' tool in Zoom for questions or comments:

Session 1: Tuesday 13 July 2021, 14:30-16:30 (Beijing time, UTC + 8); ECNU room T523

Session 2: Wednesday 14 July 2021, 19:30-21:00 (Beijing time, UTC + 8); ECNU room T523

Session 3: Saturday 17 July 2021, 21:30-23:00 (Beijing time, UTC + 8); ECNU room T523

For online participants, the ICME-14 organisers suggest the following time converter to be useful: https://www.timeanddate.com/worldclock/converter.html

For an online TSG-9 calendar (provided by the TSG-9) showing the three sessions in local time, see: https://framagenda.org/apps/calendar/p/gKSyfRDYQHCWNaiZ

Each of the three sessions features an invited 'long' paper (as per the ICME-14 guidelines) together with a number of 'short' papers. Each 'long' paper has 19 minutes, with the suggestion of 15 minutes presentation and 4 minutes for comment and questions (the presenter can vary these amounts within the 19 minutes). Each 'short' paper has 9 minutes, with the suggestion of 8 minutes presentation and 1 minutes for comment or questions of clarification (the presenter can vary these amounts within the 9 minutes). One minute is retained between presentations to allow for hand-over. The name of each presenter is in **bold** (with their institution in parentheses).

For online participation in our TSG-9, log in to the ICME-14 website to know your log in to the online conference. Then access the online platform www.icme14.org/onlinemeeting and choose Topic Study Group 9. Then log in with your log in to the online conference.

ICME-14 TSG-9

The TSG-9 Team can provide the following information on the TSG sessions:

1. One member of the TSG-9 Team, Yiling Yao (Hangzhou Normal University, China) is able to be in Shanghai at ICME-14 and to be in the TSG-9 room at ECNU for the TSG-9 sessions. Technical support from the ICME-14 organisers is also in the TSG-9 room at ECNU in Shanghai during each TSG-9 session;
2. During online participation in our TSG-9, Zoom is set for microphones to be on mute - except for the person presenting;
3. While it is standard practice at an ICME for TSG participants to *attend every session of their chosen TSG*, all presenters are asked by the ICME-14 organisers to ensure they are available *at least 15 minutes before the allotted time of their presentation*. This helps to ensure smoother transition between presentations and a better experience for participants;
4. Presenters are requested to keep to the allotted time for their presentation. TSG-9 Team member Yiling Yao (Hangzhou Normal University, China) monitors the time and reminds presenters when there is one minute remaining. There is no space for presenters to exceed their allotted time;
5. Online presenters are asked by the ICME-14 organisers to present 'live' rather than show a pre-recorded video. Pre-recorded video can be useful as back-up in case something goes wrong at the time of the presentation so there is the option to log into the ICME-14 website and upload a pre-recorded video. Such videos could be used by the technical support in Shanghai if there is a problem with a 'live' presentation;
6. As ICME-14 is an international event for which English is not the first language of the majority of participants, presentation slides with not too much text, and slow talking when presenting, helps to ensure a better experience for participants;
7. During each session and presentation, the 'chat' tool in Zoom is used for questions or comments (commencing questions or comments to a specific presenter with @name; for example @KeithJones or simply @Jones or @Keith). Questions or comments written in English are helpful to most participants. Use of Chinese, French, German, or Spanish is possible as these additional languages can translated (where appropriate) to English by the TSG-9 Team. All TSG-9 participants are invited to assist with translation into English of any other languages used in the 'chat' tool (where such translation to English would be helpful to TSG-9 participants). The TSG-9 Team hopes that good use can be made of the 'chat' tool in Zoom during TSG-9 presentations.

As stated in the ICME-14 programme 'book', TSG-9 poster contributors take an active part in our TSG-9 sessions through the 'chat' tool in Zoom for questions or comments and their posters are presented in dedicated timeslots for posters (shared by all TSGs). The timeslots are **Saturday 17 July 13:00-14:00 (Beijing time, UTC + 8)**, ECNU room **G1**, and **Sunday 18 July 13:30-14:30**, ECNU room **G1**. All TSG-9 participants are encouraged to engage with the TSG-9 poster presenters during the poster sessions. The TSG-9 poster are listed further below and can be viewed online at:
https://www.icme14.org/static/en/news/102.html?v=1625134718511

The TSG-9 Team thanks you for your participation in TSG-9 and looks forward to an enjoyable and stimulating TSG-9.

Revised 10 July 2021

TSG-9 outline agenda

Session 1: Tuesday 13 July 2021, 14:30-16:30 (Beijing time, UTC + 8)

Session Chairs: Joris Mithalal and Liping Ding (online); Yiling Yao (in Shanghai)
ECNU location **T523**
14:30-14:34
Introduction to TSG-9 and to Session 1a
Joris Mithalal (University of Lyon, France)

Session 1(a) Secondary school geometry and mathematical practices and processes (Joris Mithalal coordinating)

14:35-14:44
A teacher's use of dynamic digital technology to address students' misconceptions concerning the use of additive strategies within geometric similarity
Ali Simsek (University College London Institute of Education, UK), Celia Hoyles & Alison Clark-Wilson (9 mins)

14:45-14:54
Students spatial ability and solving-strategies for spatial geometrical, mathematical, and physical task
Marion Zoeggeler (University of Salzburg, Austria) & Guenter J. Maresch (9 mins)

14:55-15:14
Introduction of an auxiliary element as a shift of attention
Alik Palatnik (Hebrew University of Jerusalem, Israel) & Avi Sigler (19 mins, to include perhaps 4 minutes for comments and questions)

15:15-15:24
Construction program as a link between drawing and language to prepare proof process
Joris Mithalal (University of Lyon, France) (9 mins)

15:25-15:39 Session 1(a) comments, questions and conclusion
Chaired by **Joris Mithalal** (University of Lyon, France)

Session 1(b) teacher knowledge in secondary school geometry (Liping Ding coordinating)

15:40-15:44
Introduction to Session 1b
Liping Ding (Norwegian University of Science and Technology, Norway)

15:45-15:54
Understanding student teachers' mathematical knowledge for teaching geometry in a history of mathematics course
Svein Arne Sikko (Norwegian University of Science and Technology, Norway), Iveta Kohanová, Magdalini Lada, & Liping Ding (9 mins)

15:55-16:04
Teacher knowledge related to secondary school level geometry: Evidence from teacher development in SA
Jogymol Alex (Walter Sisulu University, South Africa) (9 mins)

TSG-9 outline agenda

16:05-16:14
A pre-service teacher mental structure development for understanding the geometric reflection in terms of motion and mapping view: Alexis case
Murat Akarsu (Ağrı İbrahim Çeçen University, Turkey) (9 mins)

16:15-16:24
Distinguishing content knowledge and pedagogical content knowledge for geometry teaching
Liping Ding (Norwegian University of Science and Technology, Norway) & Keith Jones (9 mins)

16:25-16:29
Session 1(b) comments, questions and conclusion
Chaired by **Liping Ding** (Norwegian University of Science and Technology, Norway)

16:30 Finish

TSG-9 outline agenda

Session 2: Wednesday 14 July 2021, 19:30-21:00 (Beijing time, UTC + 8)

Session Chairs: Matthias Ludwig (online); Yiling Yao (in Shanghai)
ECNU location **T523**
Developments in teaching secondary school geometry

19:30-19:39
Possibility of the pirates' treasure problem for teaching elementary geometry
Satoshi Takahashi, **Ryoto Hakamata** (Kochi University, Japan), & Koji Otaki (9 mins)

19:40-19:49
Inquiry-based learning using the centroids of the circumscribed equilateral triangles
Yuki Osawa (Kaichi Mirai Junior and Senior High School, Japan) (9 mins)

19:50-19:59
Study of angles and trigonometric ratio for 7th grade
Tsuyoshi Sonoda (Doshisha Junior High School, Japan) (9 mins)

20:00-20:19
Decomposing proof in secondary classrooms: a promising intervention for school geometry
Michelle Cirillo (University of Delaware, USA) (19 mins, to include perhaps 4 minutes for comments and questions)

20:20-20:29
Distance under the magnifying glass: Developing series of problems around fundamental concepts in geometry
Eszter Varga (Eötvös Loránd University, Hungary) (9 mins)

20:30-20:39
The grasp of the Pythagorean Theorem and its proof by Chinese pre-service mathematics teachers
Li Hai (Hexi University, China) (9 mins)

20:40-20:49
Implicative relationships among spatial perception, mental rotation and spatial visualisation: Implications for teaching geometry
Melih Turgut (Norwegian University of Science and Technology, Norway) & Iveta Kohanová (9 mins)

20:50-20:59
Geometry modelling outdoors with MATHCITYMAP
Matthias Ludwig (Goethe-University, Germany), Iwan Gurjanow, Simone Jablonski, & Moritz Baumann-Wehner (9 mins)

21:00 Finish

TSG-9 outline agenda

Session 3: Saturday 17 July 2021, 21:30-23:00 (Beijing time, UTC + 8)

Session Chairs: Keith Jones (online); Yiling Yao (in Shanghai)
ECNU location **T523**
Curricular issues in geometry education

21:30-21:34
Introduction to Session 3
Keith Jones (University of Southampton, UK)

21:35-21:54
Online formative assessment in geometry proving
Yael Luz (University of Haifa, Israel) & Michal Yerushalmy (19 mins, to include perhaps 4 minutes for comments and questions)

21:55-21:04
Geometric reasoning and mechanics experiment: a case study of interdisciplinary integration teaching with graphic center of gravity as an example
Feishi Gu (Shanghai Xuhui Educational Institute, China), Zhenzhen He & Liya Ban (9 mins)

22:05-22:14
A study on the performance of seventh-grade students in mathematical spatial reasoning
Zhikun Zhang (Beijing Normal University, China) & Jian Liu (9 mins)

22:15-22:24
Didactic suitability characterization of three levels of achievement on geometric drawing of secondary school students
Javier Díez-Palomar (Universitat de Barcelona, Spain) & Elvira García-Mora (9 mins)

22:25-22:34
Let's make a circle by three persons
Ken-ichi Iwase (Osaka Electro-Communication University, Japan) (9 mins)

22:35-22:44
Reconfiguration of polygons to determine the measurement of their area
Melissa Denisse Castillo Medrano (Universidad Peruana de Ciencias Aplicadas, Peru) & Jesus Victoria Flores Salazar (9 mins)

22:45-22:54
High school learners' preconceptions on the classification of quadrilaterals
Judah Paul Makonye (University of the Witwatersrand, South Africa) (9 mins)

22:55-22:59
Session 3 comments and questions, and TSG-9 conclusion
Chaired by **Keith Jones** (University of Southampton, UK)

23:00 Finish

TSG-9 outline agenda

TSG-9 posters

Saturday 17 July 13:00-14:00 (Beijing time, UTC + 8); ECNU room **G1**

Sunday 18 July 13:30-14:30; ECNU room **G1**.

The TSG-9 posters are listed below and can be viewed online at https://www.icme14.org/static/en/news/102.html?v=1625134718511

The posters are listed in the order on the ICME-14 website.

Conics: an epistemological and historical study about their geometrization
Luis Carlos Vargas Zambrano (National Polytechnic Institute, Mexico) & Gisela Montiel Espinosa

Effect of drawing solid figures based on parallel projection
Maiko Sawada (Gunma University, Japan) & Hajime Sato

"Geome-Tree": PSMTS working with tree trunks to understand circle theorems
Emmanuel Nti-Asante (University of Cape Coast, Ghana)

Promoting students deep learning by focusing on problems design: An explorative study
Minjie Chen (Ye Cheng Road School Affiliated to Nanjing Normal University, China), Jie Wang & Suhong Zhang

TSG-9 agenda (full)

Session 1: Tuesday 13 July 2021, 14:30-16:30 (Beijing time, UTC + 8)

Session Chairs: Joris Mithalal and Liping Ding (online); Yiling Yao (in Shanghai)
ECNU location **T523**

14:30-14:34
Introduction to TSG-9 and to Session 1a
Joris Mithalal (University of Lyon, France)

Session 1(a) Secondary school geometry and mathematical practices and processes (Joris Mithalal coordinating)

14:35-14:44
A teacher's use of dynamic digital technology to address students' misconceptions concerning the use of additive strategies within geometric similarity
Ali Simsek (University College London Institute of Education, UK), Celia Hoyles & Alison Clark-Wilson (9 mins)
Research has extensively documented that students develop a significant misconception associated with the incorrect use of additive strategies when engaging with geometric similarity (GS) tasks. Since dynamic digital technology (DDT) has the potential to support students to confront and reflect upon this misconception, teachers can exploit the affordances of DDT in the classroom to this goal. The aim of this paper is to explore how and why a secondary mathematics teacher uses a DDT in the classroom to promote students' understanding of why additive strategies are inappropriate to use for GS tasks. Drawing on the data collected, through classroom observations and post-lesson teacher interviews, the research findings indicate that the dynamic and visual nature of DDT can be used to help students realise the inappropriateness of the use of additive strategies for GS tasks.

14:45-14:54
Students spatial ability and solving-strategies for spatial geometrical, mathematical, and physical task
Marion Zoeggeler (University of Salzburg, Austria) & Guenter J. Maresch (9 mins)
One of the fundaments of the STEM areas are spatial abilities. Beginning from preschool, spatial ability has been shown to be significantly related to the performance in several STEM disciplines including mathematics, physics, chemistry, engineering, geology and medicine A current study at the University of Salzburg is combining three of the core fields of STEM: mathematics, geometry and physics. Two research questions of the study are: Which solving strategies (spatial and/or alternative) do students use to solve geometrical, mathematical and physical spatial tasks? Does student's spatial ability change during the years of their academic study? The results of the first question will be found with the help of special mathematical, geometrical and physic al tasks, three spatial ability tests, and with the help of interviews. A pre-test—post-test design was chosen to find answers to the second question. In the talk first results of the study are presented, information about the theoretical framework, and the connections between the understanding of several mathematic physical phenomena and spatial ability given.

TSG-9 agenda (full)

14:55-15:14
Introduction of an auxiliary element as a shift of attention
Alik Palatnik (Hebrew University of Jerusalem, Israel) & Avi Sigler (19 mins, to include perhaps 4 minutes for comments and questions)
The introduction of auxiliary elements is considered here from two perspectives: first, allowing the use of a known result or concretizing a definition in the process of problem solving and, second, as a means of shifting the focus and structure of the students' attention. We present examples and characterize the auxiliary quality of various auxiliary elements in geometric problems and proofs. Some auxiliary elements unite previously unrelated components of the original diagram; others divide a given complex entity into manageable ones. The introduction of auxiliary elements plays a dual role, enabling two types of figural operations: dimensional deconstruction and mereological division of shape. We hypothesize that there is a connection between types of figural operations and shifts of attention. Added elements become auxiliary due to the interplay of students' awareness about the heuristic reasons for their introduction and students' ability to direct attention. Implications for further educational research and mathematics instruction are proposed.

15:15-15:24
Construction program as a link between drawing and language to prepare proof process
Joris Mithalal (University of Lyon, France) (9 mins)
In this paper, we investigate how construction programs may help the coordination between drawings and language in geometry, to prepare and motivate mathematical proof. We detail a strong theoretical background about drawings, language, and proof, to identify precisely necessary and sufficient conditions for the proof process to be performed by students. Our experimental work is a continuation of previous works (Mithalal & Balacheff, 2019) in which we show that studying, correcting and writing construction programs at 4th and 5th grade progressively leads to a discursive point of view on instrument deconstruction and geometrical objects. In this perspective, we show that the didactical point of view needs to take into account discursive aspects, using tools from linguistics, as one of the issues is to learn the way geometers speak or write. We present preliminary results of a qualitative study to illustrate our theoretical hypothesis.

15:25-15:39 Session 1(a) comments, questions and conclusion
Chaired by **Joris Mithalal** (University of Lyon, France)

TSG-9 agenda (full)

Session 1(b) teacher knowledge in secondary school geometry (Liping Ding coordinating)

15:40-15:44
Introduction to Session 1b
Liping Ding (Norwegian University of Science and Technology, Norway)

15:45-15:54
Understanding student teachers' mathematical knowledge for teaching geometry in a history of mathematics course
Svein Arne Sikko (Norwegian University of Science and Technology, Norway), Iveta Kohanová, Magdalini Lada, & Liping Ding (9 mins)
In this paper we investigate Norwegian student teachers' mathematical knowledge for teaching geometry as part of a pilot study within a master course on the history and philosophy of mathematics. Attempts at writing a simple proof concerning the circumference of a hexagon inscribed in a circle was scored, and the student teachers' reflections on their proving process was analyzed. Even student teachers with a low score on the task claimed to have enhanced their knowledge and learning of mathematics. With this we contribute to the understanding of student teachers' mathematical knowledge for teaching geometry and its components.

15:55-16:04
Teacher knowledge related to secondary school level geometry: Evidence from teacher development in SA
Jogymol Alex (Walter Sisulu University, South Africa) (9 mins)
The South African secondary school geometry curriculum is relatively new and practicing teachers get trained by teacher development courses offered by tertiary institutions. This paper reports on a teacher development course in secondary school level geometry offered to practicing teachers by a tertiary institution. The sample consisted of 14 in-service mathematics teachers who voluntarily took part in the study from selected underperforming schools in one education district in a rural province in South Africa. Investigating properties of geometric figures is one of the main topics in secondary school level geometry and the practicing teachers were subjected to write a geometry test at the onset of the course. The responses to three open ended questions on reasoning and proof constituted the data for the quantitative study. Van Hiele theory and mathematical knowledge for teaching (MKfT) framework provided the lens for the analysis of the performance in the test of the sample. The lower than expected van Hiele level of geometric thinking and the low specialised content knowledge point to lack of geometry knowledge teaching in the pre-service and in-service training of teachers. The study recommends that teaching and learning of geometry in the school and pre-service level should be implemented more effectively with rich opportunities and experiences on reasoning and proof. This in turn can address the deficiencies in senior secondary school geometry instruction in South Africa.

TSG-9 agenda (full)

16:05-16:14
A pre-service teacher mental structure development for understanding the geometric reflection in terms of motion and mapping view: Alexis case
Murat Akarsu (Ağrı İbrahim Çeçen University, Turkey) (9 mins)
In this study, I describe a study of pre-service secondary mathematics teacher (PT) understanding of geometric reflection in terms of motion and mapping views. PTs often have a motion view of geometric reflection based on their understanding of the reflection line, domain, and plane. The motion view is a preliminary perspective developed prior to the construction of the mapping view, yet PTs need a mapping view of geometric reflection. However, there is no clear evidence documenting how a learner's motion view evolves to produce a mapping view. A clinical interview methodology was used to describe how mental structures occur in the movement between PT's motion view and mapping view. One case study was constructed from transcript audio records, videos, and written works. Ongoing and retrospective analyses using action, process, object and schema (APOS) framework were used to examine PT's mental structures. The results indicated that the motion view transforms into the mapping view through the development of mental structures associated with perpendicularity and equidistance properties, the role of reflection line, type of figures, operational definition of the plane.

16:15-16:24
Distinguishing content knowledge and pedagogical content knowledge for geometry teaching
Liping Ding (Norwegian University of Science and Technology, Norway) & Keith Jones (9 mins)
In this paper we examine recent studies of what teachers need to know in order to teach geometry. We raise the issue of the extent to which it is possible to distinguish CK and PCK for geometry teaching. Through closely examining two recent papers - a study by Herbst and Kosko and a study by Martinovic and Manizade - we found that it is not altogether straightforward to distinguish CK and PCK for geometry teaching. Moreover, there are alternative views by researchers of geometry CK from a more advanced mathematics perspective or from a secondary school geometry perspective. In terms of practice, the implication of our findings that is not necessarily being straightforward to distinguish CK and PCK for geometry teaching is likely to be that it is not necessarily straightforward to design education for teachers of geometry. In terms of theory, the issue of it not necessarily being straightforward to distinguish CK and PCK for geometry teaching in particular, and for mathematics teaching in general, urges more use of classroom-based research methodology, in addition to assessment items and knowledge measurement-based research, in order to have a more sufficient understand of the alternative hypothesis in research on the multidimensional or unidimensional structure of mathematical knowledge for teaching (MKT).【本文提出的研究问题是中学数学教师关于几何教学所需的数学学科知识（CK）与教学知识（PCK）在多大程度上可以加以区分。我们在本文里仔细审阅分析了最近两篇关于中学几何教学需要的专业知识的学术论文：一篇文章的作者是 Herbst 和 Kosko；另一篇的作者是 Martinovic 和 Manizade。通过分析，我们发现中学几何教学所需的数学学科知识（CK）与教学知识（PCK）的区分并非直截了当。关于中学几何教学所需的数学学科知识（CK）研究者们持有两类观点：一类是高观点下的中学几何学科知识，另一类是中学

几何教学大纲规定的学科知识。在中学数学教师师范教育的实际课程设计方面，本文的发现意味着其也同样面临关于几何教学所需的数学学科知识（CK）与教学知识（PCK）之难以区分的问题。在理论构建方面，我们的研究发现敦促应在针对数学教师专业知识的量化评估研究之外，更多的强调基于教师课堂教学实践学习的研究，以便更加充分的理解当下关于数学教师专业知识的结构的单维度或多维度的不同的研究假设。】

16:25-16:29
Session 1(b) comments, questions and conclusion
Chaired by **Liping Ding** (Norwegian University of Science and Technology, Norway)

16:30 Finish

TSG-9 agenda (full)

Session 2: Wednesday 14 July 2021, 19:30-21:00 (Beijing time, UTC + 8)

Session Chairs: Matthias Ludwig (online); Yiling Yao (in Shanghai)
ECNU location **T523**
Developments in teaching secondary school geometry

19:30-19:39
Possibility of the pirates' treasure problem for teaching elementary geometry
Satoshi Takahashi, **Ryoto Hakamata** (Kochi University, Japan), & Koji Otaki (9 mins)
The purpose of this study is to discuss the possibility of the pirate's treasure problem for teaching geometry in the perspective of inquiry-based education. This problem was originally proposed by George Gamow to demonstrate the application of complex numbers. In this paper, we report some results of a teaching experiment for college students in Japan, through analyzing the process of students' inquiry in the experiment by using the Herbartian schema--a model of inquiry offered by the anthropological theory of the didactic. In our experimentation, we observed that the student autonomously used various pieces of knowledge within elementary geometry in mathematical inquiry of the pirate's treasure problem.

19:40-19:49
Inquiry-based learning using the centroids of the circumscribed equilateral triangles
Yuki Osawa (Kaichi Mirai Junior and Senior High School, Japan) (9 mins)
This study examines a teaching material for inquiry-based learning in geometry. We introduced a new "center" of a triangle called the "CET-center", which is the centroid of a circumscribed equilateral triangle of the triangle. We proved that the locus of the CET-centers is a circle. Moreover, we gave actual lessons using the CET-center to junior and senior high school students. As a result, we concluded that the CET-center is an effective teaching material for inquiry-based learning at both junior and senior high school.

19:50-19:59
Study of angles and trigonometric ratio for 7th grade
Tsuyoshi Sonoda (Doshisha Junior High School, Japan) (9 mins)
I do 7th grade classes of geometry by using a protractor of one's own making Kaku-Shiriki and measuring the angles of everyday things around us. Learning objective are to have a broader understanding of angles, and to be introduced to trigonometric ratios. Angles are also represented in various different ways in our everyday lives. In addition to degrees, represented with the degree symbol, slopes are indicated on road signs using the percent sign % and on railway signs using the per mille sign. The values represented by %, (per mille) and the fraction all express the vertical length over the horizontal length of a right-angle triangle, i.e., the tangent. Now, we teach trigonometric ratios and functions at upper secondary school math material in Japan, but thinking about it carefully, there are many instances in which they can be used in a lower secondary school students' everyday life. They have only dealt with the special case of 45, where the vertical and horizontal lengths are 1:1. However, I think it would be possible for lower secondary students to have a sufficient understanding of the tangent by introducing it as the vertical length when the horizontal length is 1.

TSG-9 agenda (full)

20:00-20:19
Decomposing proof in secondary classrooms: a promising intervention for school geometry
Michelle Cirillo (University of Delaware, USA) (19 mins, to include perhaps 4 minutes for comments and questions)
Over three years, more than 1000 students enrolled in courses that addressed proof in secondary geometry were tested at the beginning of the school year and after completing the proof unit(s). In four of the 19 study teachers' classrooms, students were introduced to proof through a set of 16 specially designed lesson plans that addressed particular sub-goals of proof; while students in the control group learned from the standard curriculum. The sub-goals comprise a pedagogical framework that decomposes the teaching of proof in geometry. Study findings suggest that the experimental treatment had a positive statistically significant effect on students' achievement on the proof-focused post-test.

20:20-20:29
Distance under the magnifying glass: Developing series of problems around fundamental concepts in geometry
Eszter Varga (Eötvös Loránd University, Hungary) (9 mins)
The notion distance is deeply rooted in the human experience through real life situations, and hence, it is one of the instinctively most strongly founded basic geometrical ideas. At the same time, with gradual extension and generalization, it developed into a powerful, consistent cross domain mathematical concept. Students often struggle to comprehend its entirety and capture measurement methods or measures and units instead of a well-defined concept. Even textbooks fail sometimes to introduce a generalized and integrated distance notion. This contribution presents an instructional design project that relies solely on problem solving situations in provoking meaningful conversation for students better conceptual understanding.

20:30-20:39
The grasp of the Pythagorean Theorem and its proof by Chinese pre-service mathematics teachers
Li Hai (Hexi University, China) (9 mins)
A total of 215 junior high school normal students in Gansu Province, China, were tested to describe and prove the Pythagorean theorem. The statistics after the test show that the description of Pythagorean Theorem by Chinese pre-service mathematics teachers is good, more than 90% of the subjects can correctly describe Pythagorean theorem, and more subjects can correctly represent Pythagorean theorem by graphics and symbols. However, the correct rate of proving the Pythagorean theorem was lower (56.74%) , and more subjects used the string graph method of Zhao Shuang in the three kingdoms period of ancient China, the proof of Zou Yuanzhi, an ancient Chinese mathematician, and the proof of Poshjara in history of India. The errors of the participants in proving Pythagorean Theorem can be attributed to the fact that individual participants substituted narration for proof, and more participants (13.5%) substituted the establishment of special cases for the establishment of general cases, 15.35% (33 people) used the method of circular argument to prove the theorem. There were also many

participants who tried to compose the proof but did not complete the proof, and some even left blank.

20:40-20:49
Implicative relationships among spatial perception, mental rotation and spatial visualisation: Implications for teaching geometry
Melih Turgut (Norwegian University of Science and Technology, Norway) & Iveta Kohanová (9 mins)
This research aims to explore implicative relationships among spatial perception, mental rotation, and spatial visualisation to elaborate which is the core spatial ability factor. The participants are 354 undergraduate students enrolled in different departments of a faculty of education in Turkey. The data are collected through Mental Cutting Test, Purdue Spatial Visualisation Test and Mental Rotations Test, each measure different spatial ability factors. The data are analysed through an R version of Cohesion Hierarchical Implicative Classification software to produce a similarity diagram, hierarchical tree and an implicative graph regarding spatial test contexts. The findings reveal that mental rotation ability is the central spatial factor and contributes to both spatial perception and spatial visualisation and may be required for visualisation in geometry.

20:50-20:59
Geometry modelling outdoors with MATHCITYMAP
Matthias Ludwig (Goethe-University, Germany), Iwan Gurjanow, Simone Jablonski, & Moritz Baumann-Wehner (9 mins)
With the MathCityMap project (MCM) we show one possibility to use the own mobile device in a substantial and authentic learning environment. MathCityMap combines the math trail idea with the current technological possibilities of mobile devices. The MathCityMap is a two-component system. The first component is a webportal (www.mathcitymap.eu) which serves as an open access database for authentic math problems in the environment. The other component, the MCM-App, shows on a map where in the environment the problems are hidden. To solve such an authentic MCM problem you need mathematical modelling competencies. We show with the help of an examples in which way the students work with the tasks and how the system deal with different solution which occurs by solving a modelling task.

21:00 Finish

TSG-9 agenda (full)

Session 3: Saturday 17 July 2021, 21:30-23:00 (Beijing time, UTC + 8)

Session Chairs: Keith Jones (online); Yiling Yao (in Shanghai)
ECNU location **T523**
Curricular issues in geometry education

21:30-21:34
Introduction to Session 3
Keith Jones (University of Southampton, UK)

21:35-21:54
Online formative assessment in geometry proving
Yael Luz (University of Haifa, Israel) & Michal Yerushalmy (19 mins, to include perhaps 4 minutes for comments and questions)
In this paper, we report on the findings of single proving task in online interactive exploration, learning and assessment activity in geometry. We introduce a method to automatically identify and analyze common proving mistakes. We present the automatic analysis of 66 geometry students illustrated by episodes from recorded interviews of students performing the activity. Through the analysis we show various errors students made, including misuse of terms, using circular reasoning, incorrect use of deductive steps, and incomplete understanding of previously known theorems. Based on these findings, we argue that feedback drives students to use different strategies and to look for other solutions, which emphasizes the need for generating an automatic feedback on proving activity.

21:55-22:04
Geometric reasoning and mechanics experiment: a case study of interdisciplinary integration teaching with graphic center of gravity as an example
Feishi Gu (Shanghai Xuhui Educational Institute, China), Zhenzhen He & Liya Ban (9 mins)
Teaching of Interdisciplinary Integration is one of the biggest problems in new Math Curriculum Revolution in China. This article provides a class example of "the barycenter of the triangle" in Experiment of Qingpu Teaching Reform. This class is designed by the relation between logical reasoning in geometry and experimental proof in mechanics. Through the class example, this research discusses the following points: 1) Not the relation between subjects but discipline integration is an important way of break the subject limitation; 2) Using the Variant teaching in suitable knowledge point stage by stage can significantly extent students' thinking space in math problem resolving; 3) The students' activity experience in discipline integration can develop their quality of practice and innovation.

22:05-22:14
A study on the performance of seventh-grade students in mathematical spatial reasoning
Zhikun Zhang (Beijing Normal University, China) & Jian Liu (9 mins)
Spatial reasoning is a basic ability in the process of mathematics learning, it is highly relevant to students' mathematics performance in school and their future achievement in mathematics-related careers. This study is conducted on 512 seventh-grade students from 3 junior middle

schools in central region of China, a standardized mathematics test was implemented to collect students' response on spatial reasoning tasks, followed by a semi-structure task-based interview conducted on a smaller sample drawn from the test participants. Analysis of the data provided characterization of the students' performance on spatial reasoning tasks, and further development on the thinking level promoted in previous research. Results from this study could not only provide reliable reference for curriculum improvement, but also support better classroom teaching of mathematics teachers.

22:15-22:24
Didactic suitability characterization of three levels of achievement on geometric drawing of secondary school students
Javier Díez-Palomar (Universitat de Barcelona, Spain) & Elvira García-Mora (9 mins)
Teacher's practice has been able to be improved by performing analysis on student's productions and the didactical sequence related to them. In order to organize the reflection of teachers, the Onto-Semiotic Approach provides a logical sequence based on six didactic criteria which allow to identify the didactic suitability *of a didactical sequence. This research work analyses a fully written geometric task for secondary school students. The aim of the researched sequence was to apply all the contents related to geometric drawing included in the Mexican syllabus for a scholar year but by using written language and geometric notation exclusively. It was expected students would arrive to draw a complex figure made of the intersection of simple ones and the recognition of relevant points and lines as balance center or bisector. Owing to* didactic suitability *there were found the lack of some notable aspects which avoided student's success reaching the proposed geometric configuration. The absent indicators pointed out changes to make when redesigning the lesson*.

22:25-22:34
Let's make a circle by three persons
Ken-ichi Iwase (Osaka Electro-Communication University, Japan) (9 mins)
Many 3-dimentional objects around us cause interesting and mysterious problems which we might solve by using mathematical methods although we rarely notice the mysteries. For example, knots are such interesting and mysterious 3-dimensional objects and we study "teaching knot theory in a junior high-school". In this paper, we report on a lesson to solve a familiar problem on knots in our lives.

22:35-22:44
Reconfiguration of polygons to determine the measurement of their area
Melissa Denisse Castillo Medrano (Universidad Peruana de Ciencias Aplicadas, Peru) & Jesus Victoria Flores Salazar (9 mins)
In this paper, we report a study that aims to analyze the reconfiguration of secondary school students to determine the area of measurement of polygons. For this, aspects of Theory of Registers of Semiotic Representation were considered as theoretical basis. Students between 13 and 14 years of age from a private school in Lima – Peru participated in the research. We found that the students were able to solve several problems involving the calculation of the

area measurement, based on verbal and diagrammatic explanations, using the reconfiguration as operation and with the support of the squared paper.

22:45-22:54
High school learners' preconceptions on the classification of quadrilaterals
Judah Paul Makonye (University of the Witwatersrand, South Africa) (9 mins)
The study problematized learners' logical reasoning and generalisations in geometry. This reasoning is the basis of solving problems in geometry as well as geometrical proofs. This study investigated South African grade 10 high school learners' preconceptions on the classification of quadrilaterals. The Fujita (2012)'s stages for learning inclusion relationships in quadrilaterals framework informed the analysis of data gathered from fifty high school learners. Professional noticing of learner mathematical thinking (Sherin, Jacobs & Phillip, 2011) helped to gather data. The learners were given tasks on inclusion and exclusion of special quadrilateral relationships. Ten learners were selected for in-depth interviews so that their levels of geometric thinking could be more fully explained. A preliminary finding of the study is that the majority of the learners held incorrect, rigid and exclusive concept images of quadrilaterals that they were reluctant to revise during interviews.

22:55-22:59
Session 3 comments and questions, and TSG-9 conclusion
Chaired by **Keith Jones** (University of Southampton, UK)

23:00 Finish

TSG-9 agenda (full)

TSG-9 posters

Saturday 17 July 13:00-14:00 (Beijing time, UTC + 8); ECNU room **G1**

Sunday 18 July 13:30-14:30; ECNU room **G1**.

The TSG-9 posters are listed below and can be viewed online at https://www.icme14.org/static/en/news/102.html?v=1625134718511

The posters are listed in the order on the ICME-14 website.

Conics: an epistemological and historical study about their geometrization
Luis Carlos Vargas Zambrano (National Polytechnic Institute, Mexico) & Gisela Montiel Espinosa

Effect of drawing solid figures based on parallel projection
Maiko Sawada (Gunma University, Japan) & Hajime Sato

"Geome-Tree": PSMTS working with tree trunks to understand circle theorems
Emmanuel Nti-Asante (University of Cape Coast, Ghana)

Promoting students deep learning by focusing on problems design: An explorative study
Minjie Chen (Ye Cheng Road School Affiliated to Nanjing Normal University, China), Jie Wang & Suhong Zhang