

## TSG Agenda

### TSG 15: Teaching and Learning Discrete Mathematics

Class: A

\*\*Please prioritize the sessions in “core-time” (from 19:30-23:00, Beijing time, i.e. Session 2, 3 for Class A and session 1, 2 for Class B) as they are friendly to most of the time zones in the world.

#### Overview:

#### Session 1: 7/13/21 14:30-16:30 Beijing Time

14:30-14:50	Borys	Suggestions for an integration of cryptology into a math curriculum
14:50-15:10	Ouvrier- Buffet	Enriching pre-service teachers' conceptions about proof with discrete mathematics
15:10-15:30	Windler	Graph theory in primary school mathematical education - a quantitative study on the impact of graph theory concepts on psychological characteristics of fourth grade students
15:30-15:50	Silva	The role of discrete mathematics in secondary mathematics for non-stem paths
15:50-16:10	Gosztonyi	discrete mathematics in the hungarian mathematics curriculum
16:10-16:30	Balaji	Discrete mathematics into K-11 and K-12 Grade Education

#### Session 2: 7/14/21 19:30-21:00 Beijing Time

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19:30-20:00	Tillema	leveraging combinatorial and quantitative reasoning to support the generaliation of advanced algebraic identities
20:00-20:20	Hoeverler	combinatorial counting problems in elementary school: a comparative analysis of german textbooks
20:20-20:40	Antonides	case studies on the combinatorial reasoning of future middle school teachers

20:40-21:00	Mota	guiding students' reinvention of combinatorial operations
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**Session 3: 7/17/21 21:30-23:00 Beijing time**

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21:30-21:50	Lockwood	preservice teachers' development of mathematical knowledge for teaching via combinatorial tasks in a computational setting
21:50-22:10	Medova	Relation between computational and combinatorial thinking of undergraduate students of applied informatics
22:10-22:30	Durcheva	Some approaches for incorporation of CAS in a discrete mathematics course
22:30-22:50	Stettner	how can poly-universe sets develop creativity during the solution of combinatorial exercises
22:50-23:00	Wrap up and discussion	

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**Session 1**

1. Time: 14:30-14:50

Title of the Paper: Suggestion for an Integration of Cryptology into a Math Curriculum

Author(s): Dr. Thomas Borys

Institution(s) (to school/department/research center) and Country/Region: University of Education Karlsruhe, Institute of Mathematics, Germany

Short abstract of the paper (20 lines maximum): Cryptology is a very old science and just a few decades ago it was a science for government, military, secret services and spies. Nowadays, cryptology is almost everywhere in our lives. Many cryptologic applications are using discrete math especially all the newer ones. Thus discrete math is always around us. This paper reports on a research to integrate cryptology as a special topic of discrete math into a standard math curriculum. In a first step fundamental ideas of mathematics

which are the basic guidelines for mathematical education are discussed. In a second step connections between the set of fundamental ideas and various techniques of cryptology are shown. One outstanding example with high invitingly character for this is the Diffie-Hellman key exchange. Finally all deliberations are fixed in a curriculum.

2. Time: 14:50-15:10

Title of the Paper: Enriching Pre-Service Teachers Conceptions About Proof with Discrete Mathematics

Author(s): Cecile Ouvrier-Bufferet

Institution(s) (to school/department/research center) and Country/Region: Universite Paris-Est Creteil, France

Short abstract of the paper (20 lines maximum): The following article proposes an opening of a well-known situation coming from the theory of didactical situation (namely The Race to 20), leading to an ongoing open problem for mathematicians. This discrete situation is at the interplay of game theory, number theory and algorithmics. Its use in a pre-service secondary teachers training course aims at encouraging teachers to reflect on the mathematical activity of proof and its writing in the classroom.

3. Time: 15:10-15:30

Title of the Paper: Graph theory in primary school mathematical education - a quantitative study on the impact of graph theory concepts on psychological characteristics of fourth grade students

Author(s): Dr. Melissa Windler

Institution(s) (to school/department/research center) and Country/Region: University of Bremen, Germany

Short abstract of the paper (20 lines maximum): The question of suitable teaching content in order to provide students with a kind of motivation to learn, perform, and prepare them for their present and future lives is essential for mathematical education. In the course of the current digital transformation and in view of new application possibilities, topics of discrete mathematics, especially graph theory, could become very important for mathematics teaching as well as for the increasingly demanded school subject of computer science in primary schools. For this reason, a quasi-experimental intervention study in Germany investigated the extent to which a specially developed teaching unit on graph theory in primary school mathematics influences motivation, self-concept, attitudes towards mathematics, and the mathematical performance of fourth-grade students. The results show that in the classes in which the graph theory lesson was conducted there was an increase in self-concept, attitude, and mathematical achievement. These results are used to derive and discuss new insights for the design of mathematical education.

4. Time: 15:30-15:50

Title of the Paper: The Role of Discrete Mathematics in Secondary Mathematics for Non-Stem Paths

Author(s): Jaime Carvalho e Silva

Institution(s) (to school/department/research center) and Country/Region: Universidade de Coimbra, Portugal

Short abstract of the paper (20 lines maximum): Mathematics becomes more and more important at the Secondary School level. Which mathematics should they study? More Algebra and Geometry? The Calculus sequence? Statistics? Discrete Mathematics? Or a mix of them all? It seems that Discrete Mathematics is becoming increasingly important. We discuss what has been done in a few countries. We concentrate on the experience of a new course in Portugal developed mainly around Discrete Mathematics and Statistics.

5. Time: 15:50-16:10

Title of the Paper: Discrete Mathematics in the Hungarian Mathematics Curriculum

Author(s): **Katalin Gosztonyi** and Csaba Csapodi

Institution(s) (to school/department/research center) and Country/Region: Eötvös Loránd University of Budapest, Mathematics Teaching and Education Centre, Hungary

Short abstract of the paper (20 lines maximum): Discrete mathematics is traditionally an important domain of Hungarian mathematics and occupies also a significant place in the Hungarian curricula. In this paper, we first briefly discuss the appearance of discrete mathematics in Hungarian mathematical research. Then, we shortly treat Tamás Varga's curricular reform project, where the discrete mathematics obtained an important place in the general education curricula. In the last part, we present the role of discrete mathematics in the current curriculum, focusing especially on the domain of combinatorics.

6. Time: 16:10-16:30

Title of the Paper: Discrete Mathematics into K-11 and K-12 Grade Education

Author: Balaji N & Karthik Pai B H

Institution(s) (to school/department/research center) and Country/Region: Department of Information Science and Engineering, NMAM Institute of Technology, Nitte, Karakala – Taluk, Udipi, Karnataka, India

Short abstract of the paper (20 lines maximum): Discrete Mathematics course is a core part of Computer or Information Science and Technology and it facilitates the study of applications in the field of Computer Science. In this article, we are glancing the syllabi of K-11 and K-12 grade education. We look into the recent situation on K-11 and K-12 regarding discrete mathematics. Then we discuss the reasons why to teach about it as well as specific educational targets. To establish results by counting a certain collection of objects provides the combinatorial identities; and also provides a novel proof techniques. We have described the significant role of discrete mathematics in K-11 and K-12 college classrooms. We illustrate shortly the main targets specific for discrete mathematics education, as we designed so far. We have examined the current situation, how teachers are conducting the different topics of study in discrete mathematics and its applications perspective. We believe that showing it to them the application of discrete mathematics

would improve their studies. They would be able to nurture themselves and use their discrete mathematics related knowledge and skills in new areas.

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## Session 2

1. Time: 19:30-20:00

Title of the Paper: Leveraging Combinatorial and Quantitative Reasoning to Support the Generalization of Advanced Algebraic Identities

Author(s): **Erik S. Tillema** and Lori Burch

Institution(s) (to school/department/research center) and Country/Region: Indiana University, United States of America

Short abstract of the paper (20 lines maximum): In this paper we outline a combinatorial approach to developing advanced algebraic identities that we have used with secondary students and teachers. We focus on identities that involve non-linear multiplicative relationships, and are the basis for understanding many properties of polynomials (e.g.,  $(ax)^2 = a^2 x^2$  or  $(x+a)(x+b)(x+c) = x^3 + (a+b+c)x^2 + (ab+ac+bc)x + abc$ ). We situate our approach in relation to Thompsons (2011) work on quantitative reasoning and Kaputs (2008) framework on algebraic reasoning. In doing so, we argue that many current algebra curricula do not aim to support students to generalize advanced algebraic identities based on combinatorial structure, and that a lack of focus on combinatorial structure increases the likelihood of supporting less powerful forms of generalization. We add this use of combinatorial reasoning to a long-standing body of research suggesting its importance to a range of mathematical domains (e.g., Kapur, 1970)

2. Time: 20:00-20:20

Title of the Paper: Combinatorial Counting problems in elementary school: A comparative analysis of German textbooks

Author(s): **Karina Höveler** and Janet Winzen

Institution(s) (to school/department/research center) and Country/Region: Westfälische Wilhelms-Universität Münster, Germany

Short abstract of the paper (20 lines maximum): Unlike in some other countries combinatorial counting problems are mandatory curriculum content in German elementary schools since 2004. Based on empirical studies there are recommendations which contents should be addressed in teaching practice and how they should be addressed. But yet little is known about what and how combinatorial counting is taught. For this purpose, a study on combinatorial textbook contents and teaching practices was conducted. This paper presents results of the comparative analysis of ten elementary mathematics textbooks and the corresponding teacher handouts. With regard to counting strategies and principles it points out that empirical-based recommendations on meaningful content-related priorities are given little consideration.

3. Time: 20:20-20:40

Title of the Paper: Preliminary Levels of Sophistication for Enumerating Permutations

Author(s): **Joseph Antonides** and Michael T. Battista

Institution(s) (to school/department/research center) and Country/Region: Ohio State University, USA

Short abstract of the paper (20 lines maximum): Two preservice middle-school teachers (PSTs) each participated in a one-on-one constructivist teaching experiment with the goal of capturing each PST's developing constructions of combinatorial concepts and operations in contexts of interactive mathematical communication. The data reported here focus on the PSTs' constructive processes regarding permutations, represented as n-cube towers with n different colors of cubes in each tower. We present a preliminary set of levels of sophistication in the PSTs' forms of reasoning about permutations, as inferred from our analysis; at the final levels proposed in this report, students construct a

generalized concept of factorial and can apply this concept to reason about combinatorial enumeration in novel circumstances.

4. Time: 20:40-21:00

Title of the Paper: Guiding students' reinvention of combinatorial operations

Author(s): **Belmira Mota** 1,2 and Rosa Antónia Tomás Ferreira 2,3

Institution(s) (to school/department/research center) and Country/Region: 1) Colégio Efanor, 2) Faculdade de Ciências da Universidade do Porto 3) CMUP CMUP , Portugal

Short abstract of the paper (20 lines maximum): In this paper, we share the results of a teaching experiment involving a class of thirty-one 12<sup>th</sup> graders (17-18 years-old), in northern Portugal, for 12 lessons of 90 minutes each, aimed at engaging them in the guided reinvention of the formulas for the basic combinatorial operations, focusing on combinations. Supported on the premises of Realistic Mathematics Education and on Lockwood's model for students' combinatorial thinking. Students worked in challenging and contextualized tasks in an inquiry-based teaching approach. Data were collected through direct observation, videotaped lessons and several documents. The results corroborate the relationships described in Lockwood's model. Yet, a strong relationship seems to exist between counting processes and the formulas/expressions that translate the answers to the tasks. We suggest some aspects that need to be accounted for in the design of tasks, searching to support a deeper understanding of the formulas for combinations, and we offer some thoughts on possible details that may be added to Lockwood's model.

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### Session 3

1. Time: 21:30-21:50

Title of the Paper: Preservice Teachers' Development of Mathematical Knowledge for Teaching via Combinatorial Tasks in a Computational Setting

Author(s): **Elise Lockwood** and Adaline De Chenne

Institution(s) (to school/department/research center) and Country/Region: Oregon State University, United States of America

Short abstract of the paper (20 lines maximum): In this paper, we share findings from a design study in which we created a set of combinatorial tasks in a computational setting involving basic Python programming. We report on results of implementing these tasks in a course for pre-service mathematics teachers in the United States. We exemplify one way in which our tasks seemed to reinforce important aspects of pre-service teachers' mathematical knowledge, particularly their knowledge of content and students in relation to addition and multiplication. Our findings suggest that leveraging combinatorial tasks in a computational setting could be a productive way to highlight certain aspects of mathematical knowledge for teaching.

2. Time: 21:50-22:10

Title of the Paper: Relation between Algorithmic and Combinatorial Thinking of Undergraduate Students of Applied Informatics

Author(s): **Janka Medová** and Soňa Čeretková

Institution(s) (to school/department/research center) and Country/Region: Department of Mathematics, Faculty on Natural Sciences, Constantine the Philosopher University in Nitra, Slovakia

Short abstract of the paper (20 lines maximum): Mathematicians use computing in their work regularly. One of the stages of the years-lasting development of the ability to solve combinatorial problems is the use of generative strategies for listing all the elements of set of outcomes. The generative strategies can be considered as algorithms leading to listing all the elements. The relation between the coding abilities in combinatorics and

ability to solve the counting and reasoning problems in combinatorics was quantitatively assessed. The solutions of the three sets of problems submitted by 45 undergraduate students were submitted to analyses. The solutions were divided into the two groups according the correctness of the solution of the coding problem. The difference in solving enumeration/counting problems between the two groups was not significant. On the other hand, the difference in reasoning task was significant for problems with all three implicit combinatorial models (distribution, selection, partition).

3. Time: 22:10-22:30

Title of the Paper: Some Approaches for Incorporation of CAS in a Discrete Mathematics Course

Author(s): Mariana Durcheva

Institution(s) (to school/department/research center) and Country/Region: Technical University of Sofia, Bulgaria

Short abstract of the paper (20 lines maximum): Computer algebra systems (CAS) provide the teacher with many opportunities for new teaching methods, as well as enhance active learning of students. The Discrete Mathematics (DM) course includes topics giving students the knowledge and skills needed for programming and computer science courses. This work discusses different approaches for using CAS in a DM course.

4. Time: 22:30-22:50

Title of the Paper: How Can Poly-Universe Sets Develop Creativity During the Solution of Combinatorial Exercises?

Author(s): **Eleonóra Stettner** and Szabina Tóth

Institution(s) (to school/department/research center) and Country/Region: Hungarian University of Agriculture and Life Sciences, Szabó Lőrinc Bilingual Primary and Secondary School, Hungary

Short abstract of the paper (20 lines maximum): In 2018-2019 under an Erasmus+ project we (mathematics teachers) had an opportunity to think about the Poly-Universe game - created by fine artist János Szász SAXON – from a mathematical aspect together with our students. We found connections with many spheres of mathematics, however, this game can develop combinatorial thinking in particular. We, teachers from 4 countries created PUSE (Poly-Universe in School Education) Methodology which is a collection of about 150 exercises. In our opinion these activities reflect the approach and attitude of teachers from different countries. This is what makes this book special and different from other activity collections and the fact that the activities are related to a tangible game which can be used to construct the given problem and find an idea to the solution of the task.

## **WRAP UP AND DISCUSSION 22:50-23:00**

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### **Note:**

#### Class A:

- Session 1: 14:30-16:30 Beijing time, July 13<sup>th</sup>
- Session 2: 19:30-21:00 Beijing time, July 14<sup>th</sup>
- Session 3: 21:30-23:00 Beijing time, July 17<sup>th</sup>

#### Class B:

- Session 1: 19:30-21:00 Beijing time, July 13<sup>th</sup>
- Session 2: 21:30-23:00 Beijing time, July 16<sup>th</sup>
- Session 3: 14:30-16:30 Beijing time, July 17<sup>th</sup>