# INTUITIVE UNDERSTANDING OF INFINITE GEOMETRIC SERIES CONVERGENCE VALUES FOR STUDENT SUPPORT

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## 1. INTRODUCTION

**Most students can find the value by calculating the convergence value of an infinite geometric series.**

**However, even students who are good at mathematics, even if it is understood by calculation ,cannot intuitively understand why the convergence value of an infinite geometric series is that value and why it actually converges.**

**I have created teaching materials to intuitively understand the convergence values of infinite geometric series.**

**It is an easy way to calculate the proportions of a tape diagram.**

**Logic is important , especially for students studying mathematics , but it is more important to be able to imagine without being particular about the details.**

## 2. PREPARATION(LEMMA)

Fig. 1

Three similar triangles

a : b = c : d
= a + c : b + d

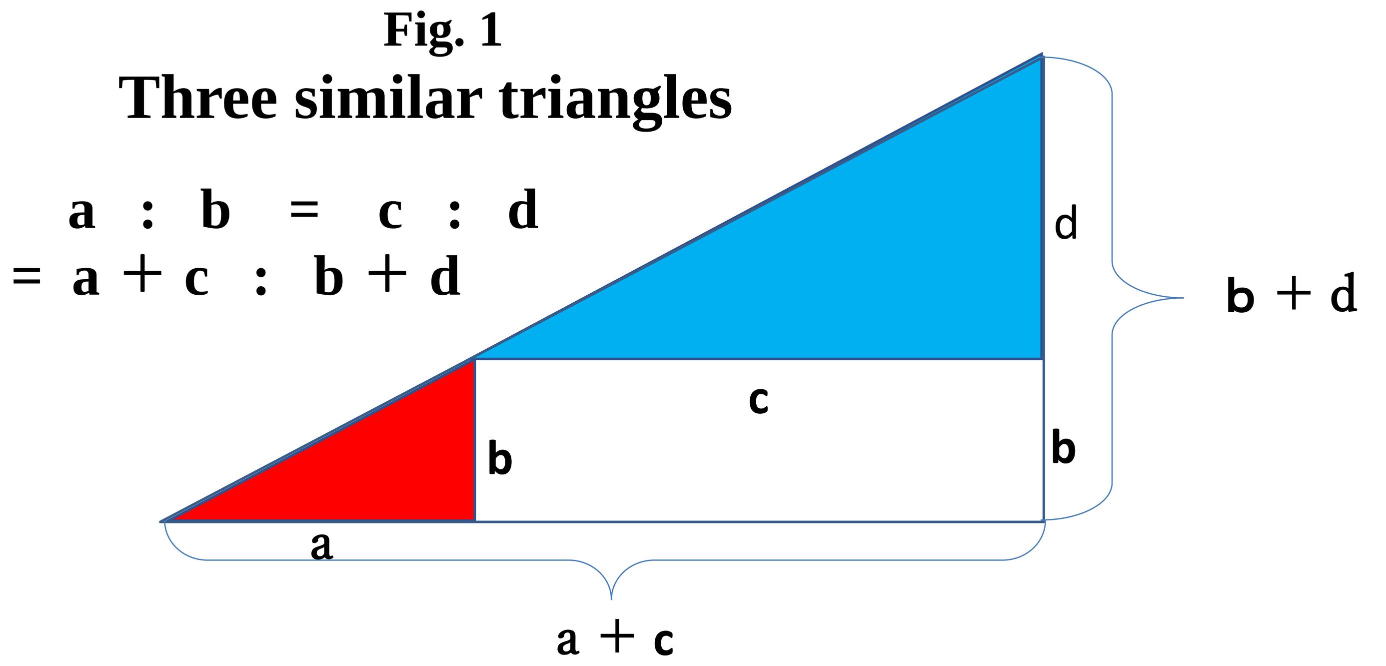


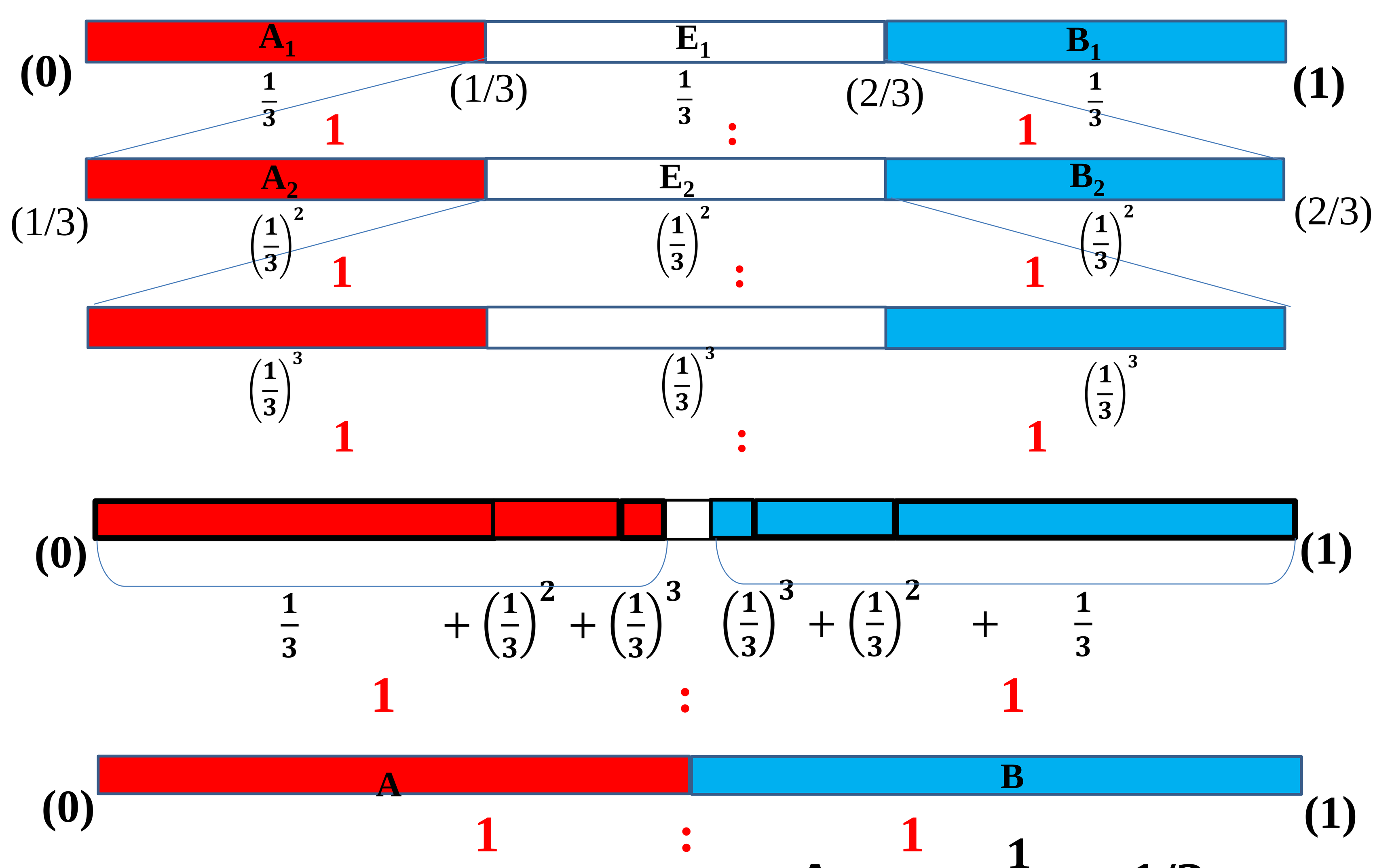

**Fig. 1** derives the following equation. componendo

$$a_1 : b_1 = a_2 : b_2 = \cdots = a_n : b_n$$
$$= a_1 + a_2 + \cdots + a_n : b_1 + b_2 + \cdots + b_n$$

## 3. EXAMPLE of this method

### (ex.1) $\frac{1}{3} + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3 + \left(\frac{1}{3}\right)^4 + \cdots = A$

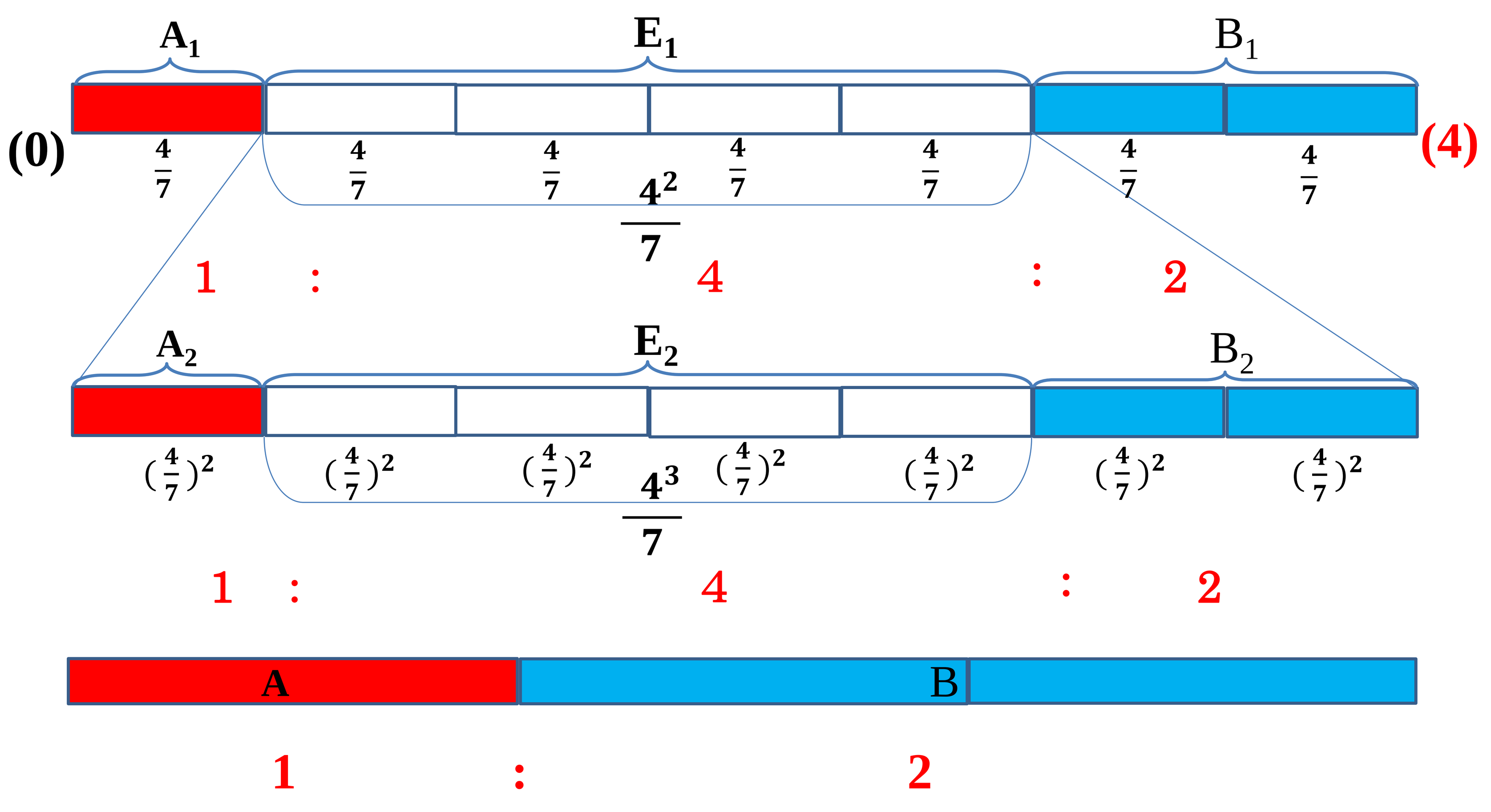
Divide interval [0,1] into three intervals $A_1$, $E_1$, $B_1$ of equal length. (from left to right) .
Length of $A_1$ is abbreviated as $(A_1)$ .So, $(A_1) : (B_1) = 1 : 1$
Similarly, divide interval $E_1$ into three intervals $A_2$, $E_2$, $B_2$ of equal length.
Then,$(A_2) : (B_2) = 1 : 1$. Repeat this operation. Length of $E_n \to 0$.
Therefore, interval [0,1] is finally as shown in the diagram below.




$$\therefore \quad A = \frac{1}{1+1} = 1/2.$$

### (ex.2) $\frac{4}{7} + \left(\frac{4}{7}\right)^2 + \left(\frac{4}{7}\right)^3 + \cdots = A$ $r = 4/7$ (> 1/2).

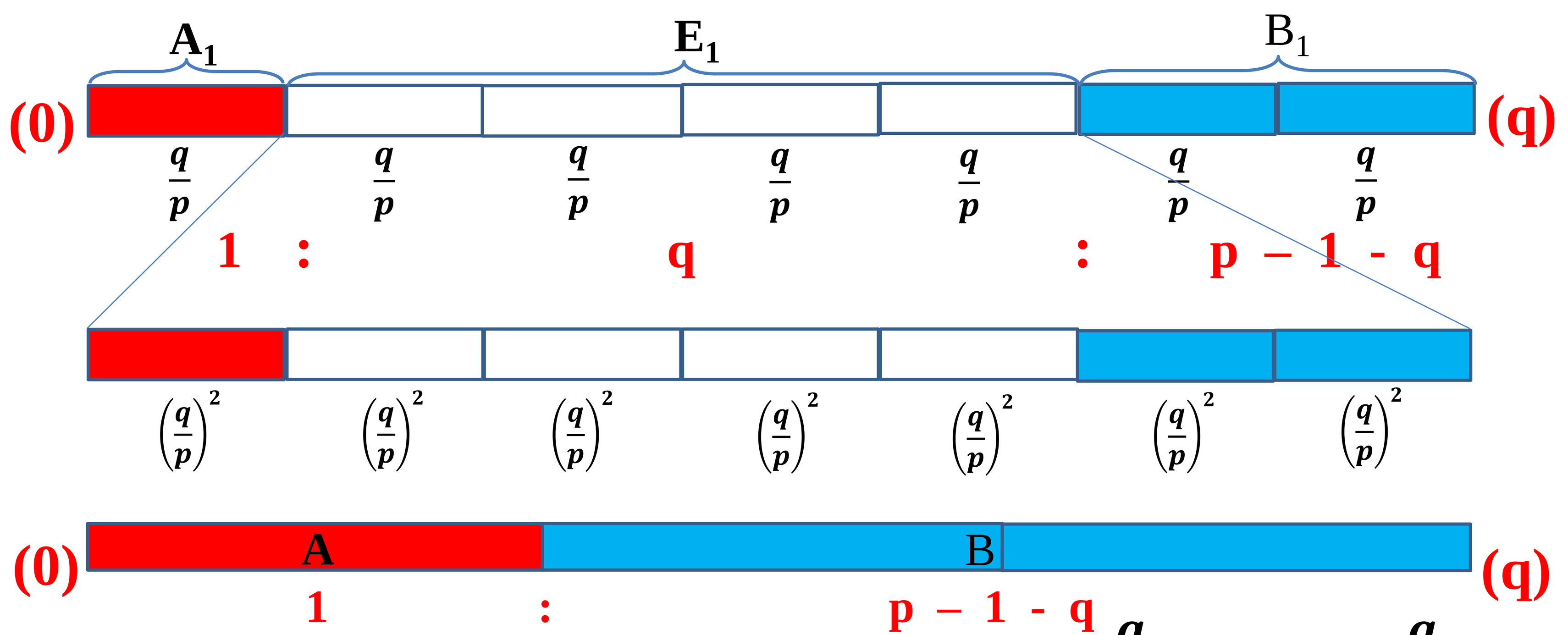
Divide interval **[0,4]** into three intervals $A_1$, $E_1$, $B_1$ . $(A_1) : (E_1) : (B_1) = 1 : 4 : 2$
Similarly, divide interval $E_1$ into three intervals $A_2$, $E_2$, $B_2$ .
And, $(A_2) : (E_2) : (B_2) = 1 : 4 : 2$ . Repeat this operation. Length of $E_n \to 0$.
Therefore, interval **[0,4]** is finally as shown in the diagram below.




$$\therefore \quad A = \frac{4}{7} + \left(\frac{4}{7}\right)^2 + \left(\frac{4}{7}\right)^3 + \cdots = \frac{4}{1+2} = 4/3$$

### (ex.3) $\frac{q}{p} + \left(\frac{q}{p}\right)^2 + \left(\frac{q}{p}\right)^3 + \cdots = A$ (0<p<q;integer)

Divide interval **[0,q]** into three intervals $A_1$, $E_1$, $B_1$ . $(A_1) : (E_1) : (B_1) = 1 : q : p-1-q$ .
Repeat this operation. Length of $E_n \to 0$.
Therefore, interval **[0,q]** is finally as shown in the diagram below.

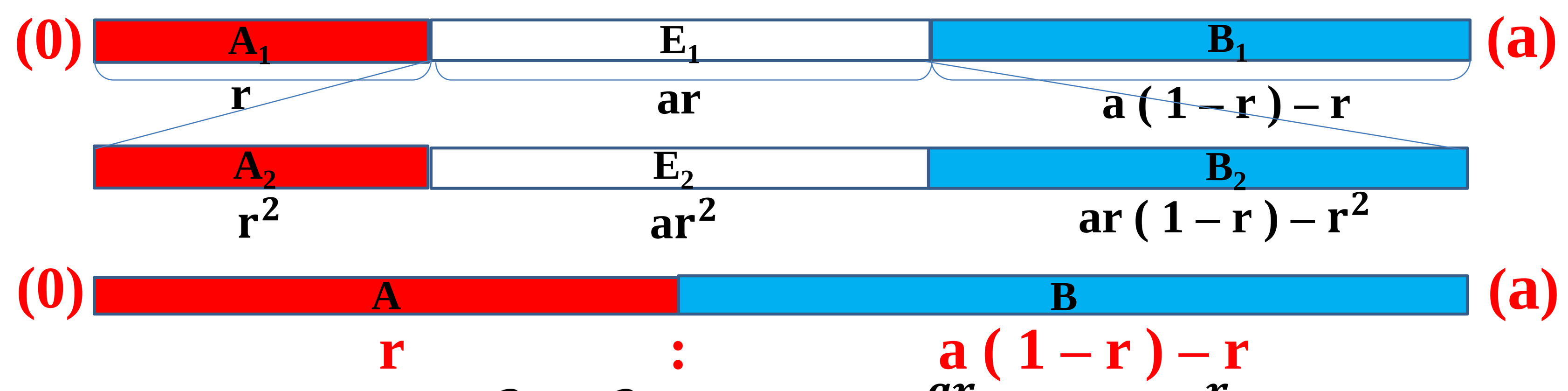



$$\therefore \quad A = \frac{q}{1 + p - 1 - q} = \frac{q}{p - q}$$

## 4. SUMMARY

**This method can be extended if the value of r is a real number in the interval (-1,1).**

Divide interval **[0,a]** into three intervals $A_1$, $E_1$, $B_1$ .(0 < r <1,r < a:real number)
$(A_n) : (E_n) : (B_n)$ (n=1,2,3,・・) = $r : ar : a(1-r) - r$ . Length of $E_n \to 0$.




$$\therefore \quad A = r + r^2 + r^3 + \cdots = \frac{ar}{r + a(1-r) - r} = \frac{r}{1-r}$$

$-1 < r < 0$ **: Since the sign of the term changes alternately, the value of the partial sum moves left and right alternately in the tape diagram. You can do the same.**

**References**

MATSUO.yoshitomo. A Study Models in Mathematics Education.

*Arithmetic and Mathematics Education Practice 20, Arithmetic and Mathematics Education Practice Lecture Publication(1988),* 406–410.