

# ICME-14, July 11–18, 2021

## TSG 13: Teaching and Learning of Calculus

### Agenda, Class A

session	date	China Standard Time	Central European Summer Time	Coordinated Universal Time (UTC)	Eastern Daylight Savings Time
1	13-Jul	14:30–16:30	8:30–10:30	6:30–8:30	2:30–4:30 am
2	14-Jul	19:30–21:00	13:30–15:00	11:30–13:00	7:30–9:00 am
3	17-Jul	21:30–23:00	15:30–17:00	13:30–15:00	9:30–11:00 am

All times listed in the agenda are China Standard Time (UTC + 8 hours)

#### Session 1, 13 July

1. Time: 14:30–14:45

#### **Mathematical Knowledge for Teaching of Calculus: An Exploratory Study of Secondary School Teachers Mathematical Thinking Related to Concepts in Calculus**

**Jonaki B Ghosh**

Lady Shri Ram College for Women, University of Delhi, New Delhi, India

This article describes a study which was undertaken to explore the nature of mathematical knowledge used by senior secondary school teachers while teaching the fundamental concepts of Calculus. Based on the MKT framework, 16 items were designed to elicit Specialised Content Knowledge (SCK) and Knowledge of Content and Students (KCS) of teachers related to the concepts of limits, continuity and derivatives. Teachers responses to these items were analysed and some interesting facts were revealed with regard to their understanding of these concepts, their pedagogical approaches and their thinking related to students conceptions or misconceptions.

2. Time: 14:50–15:05

#### **MODELING CONCEPTS OF DERIVATIVE AND DIFFERENTIAL WITH EDUCATIONAL SOFTWARE**

**Vladimir Nodelman**

Holon Institute of Technology, Holon, Israel

Learning new fundamental concepts is greatly facilitated by the use of visual models. These models should be isomorphic to the studied contents and simple to percept (Boltyansky, 1970). The proper students' activities with models serve the knowledge interiorization. The report offers system of such modeling for studying concepts of derivative and differential through the authors nonprofit software VisuMatica.

3. Time: 15:10–15:25

### **CONSTRUCTING KNOWLEDGE USING DIGITAL TOOLS: THE CASE OF THE INFLECTION POINT**

**Regina Ovodenko**, Anatoli Kouropatov

Shenkar College of Engineering , Netanya, Israel

This paper reports part of the study conducted on a pair of first-year students from the Industrial Engineering College. The students learned the notion of inflection point using a digital unit that has been designed specifically for that purpose. We use the theory of Abstraction in Context (Hershkowitz, Schwarz, & Dreyfus, 2001) as a theoretical framework and as a methodological tool to analyze the students learning process. We found some evidence that learning with this unit would allow students to confront errors and to (perhaps partially) construct knowledge about the inflection point.

4. Time: 15:30–15:45

### **STUDENTS INTERPRETATIONS OF THE DEFINITE INTEGRAL**

**Inen Akrouti**

Virtual University of Tunis, Bizerta, Tunisia

This paper focuses on students interpretations of the definite integral in the first year of studying science in university. Data were collected from students written responses of questions that relate to their views of integration. My analysis is based on prior researches that had shown different students conceptualisation of definite integral (Jones, 2015a, 2015b). Analysis of results identified three categorizations for students interpretations of the definite integral: area, antiderivative and limit of approximations. Similarly, when asked about computing an integral in non-routine formula, students responses were related to two process: those based on area process, and those based on antiderivative process. It is important to notify that, although a majority of students have interpreted definite integral in terms of area underneath a curve; few numbers of them have used area to evaluate the given integral in question Q2. They choose the algebraic process of evaluating a definite integral using the Fundamental Theorem of Calculus. Participants were first-semester calculus students enrolled in a public university.

5. Time: 15:50–15:55

## **COMPARISON OF MATHEMATICS TEXTBOOKS IN IB SCHOOL AND CHINESE PUBLIC HIGH SCHOOL: TAKE CORE CONCEPT--CALCULUS AS AN EXAMPLE**

**Yun Lu**

Education department of East China Normal University, China

This study compares core concept of calculus in the mathematics textbooks of IB schools and Chinese high schools. One of the selected textbooks is the peoples education press edition A and the other is IBDP mathematics textbook called Mathematics for the international student mathematical HL (core).third edition published by Haese and Harris publications. Based on the theory of cognitive psychology, this paper analyzes the similarities and differences of the hierarchical structure of calculus in these two textbooks through the drawing of concept map, and studies the presentation of the lower concept of calculus -- definite integral through text analysis and cognitive level analysis framework, aimed to provide some Suggestions for the compilation of high school mathematics textbooks in the new curriculum reform in China.

6. Time: 15:58–16:03

## **RESEARCH IN CALCULATING AREAS BETWEEN CURVES**

**Gordana Stankov**, Djurdjica Takači

Subotica Tech College of Applied Sciences, Novi Knezevac, Serbia; University of Novi Sad, Novi Sad, Serbia

This paper summarizes the experiences in learning how to calculate areas between curves using computer at Subotica Tech College of Applied Sciences, Serbia. We compare the way of learning and the learning achievement of the experimental group (where students learned individually using computer) and the control group (where students learned individually without the use of computer).

7. Time: 16:06–16:11

## **Cause Analysis And Solutions on The Problems in Teaching The Concept of Differential**

**Yingzhe Ban**, Qi Zhang

Peking University, Beijing, China; Beijing Normal University, Beijing, China

Owing to historical reasons, how to understand the essence of differential has long been in dispute and it is admitted that there do exist difficulties in teaching the concept of differential. We investigated various definitions of differential among almost all of the textbooks used in Mainland China and concluded that these definitions are actually faulted in view of formal logic. Also, many calculus teachers have tried to reform their methods of teaching calculus and we referred to some methods to solve the problems.

8. Time: 16:14–16:19

## **THE CONCEPT OF CONTINUITY THROUGH DIFFERENT TYPES OF REPRESENTATIONS OF THE FUNCTION**

**Matthaios Antonopoulos**, Eleonora Antonopoulou

University of Athens, Nea Pedeli, Greece

Continuity as a concept has a central role in mathematics. This research was carried out on students of mathematics. The purpose of this work is to study the treatment of students when controlling the continuity according to the type of representation of the function. The results of the research show that there are often differences in the way in which the continuity is characterized and some students' responses show contradictions depending on the representation of the function.

9. Time: 16:22–16:27

### **How to Teach Calculus Correctly**

**Han Yue**

In this paper, firstly, on the basis of sufficient evidences and logical analyses, we point out the systematic faults and shortcomings in the current principles of calculus; secondly, we give the teaching method of a transitional system of principles of calculus, which is an expediency; thirdly, we emphasize that the new definition of the differential given in this paper isn't perfect, however, we need to rewrite the textbook of calculus now.

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### **Session 2, 14 July**

1. Time: 19:30–19:45

### **ACTIONS IN THE LEARNING ENVIRONMENT: ANALYZING PHYSICS AND MATHEMATICS LESSONS IN THE CASE OF ODE**

**Kristina Elisabeth Juter**, Örjan Hansson, Andreas Redfors

Faculty of education, Kristianstad University, Kristianstad, Sweden

This study was conducted to investigate what different and similar types of communication a teacher has with his students in a physics class and a mathematics class (with partly the same students); during both lessons, models containing ordinary differential equations (ODE) were treated. The main aim of this paper is to investigate how two theoretical frameworks can be used to jointly provide information about the lessons implementation, one of which focuses on actions in the learning environment, while the other focuses on the character of conversations in terms of mathematics, theoretical models and reality. Episodes of

conversations were identified for both classes and categorized in accordance with the frameworks. The two frameworks proved to complement each other well in the analysis of the lessons and enabled an informative analysis of the teachers actions for the students learning during their respective mathematics and physics lessons.

2. Time: 19:50–20:05

### **From upper secondary school to university calculus: Language difficulties versus conceptual difficulties**

**Arne Hole**, Inger Christin Borge, Liv Sissel Grønmo

University of Oslo, Oslo, Norway

We take our point of departure in a study of scores on some problems taken from the autumn 2015 exam in the first calculus course MAT1100 at the University of Oslo. Results indicate that the students who failed the exam, scored significantly better on new topics introduced in the course than they did on a basic algebraic task covered in the curriculum of the first year of Norwegian upper secondary school. In the light of these results, we discuss the importance of working systematically with the comprehension of abstract mathematical language in calculus courses. We argue that in many cases, learning obstacles experienced by calculus students may be related to the use of variables and other ingredients of mathematical language, instead of being related to comprehension of particular concepts of calculus such as limit, derivative and so on. It is argued that common frameworks for mathematical competencies may fail to grasp the essence of these language problems, which are related to the process of abstraction.

3. Time: 20:10–20:15

### **THE DISCRETE-DENSE-CONTINUOUS PHENOMENON AND ITS IMPLICATION IN CONTINUOUS**

**Elizabeth Montoya Delgadillo**

Pontificia Universidad Catolica de Valparaiso, Chile

This paper deals with the discreteness-density-completeness triad in analysis for in- and pre-service mathematic teachers. It is an important issue for teaching and learning analysis, besides it remains implicit for their students. It is for instance the case when linking points on a graphic to produce a continuous curve. With this example, we present the evolution of the situations in teachers training we designed

4. Time 20:18–20:23

### **A LIMIT FREE CALCULUS FOR INTRODUCING THE CONCEPTS OF TANGENT AND ASYMPTOTE. AN EDUCATIONAL PROPOSAL INSPIRED BY THE PAST**

**Maria Astrid Cuida Gomez**, Edgar Martínez-Moro, Héctor Sanz Herranz

Universidad de Valladolid, Valladolid, Spain

Nowadays the concept of limit in certain advanced secondary school courses covers more than two thirds of the classroom and student time when learning calculus. In this work we inquire ourselves what is the real purpose for introducing limits in this type of courses and, if the "final product" of understanding the derivative could be accomplished better without introducing the concept of limit or at least doing it later. In order to accomplish this we explore the geometric and algebraic procedures used in the past for computing the tangent line and asymptote to a (rational) curve and adapt them to the actual geometric and algebraic language as well as to the use of CAS in teaching. As a conclusion we may say that limits buried the understanding of derivatives as it was also that Cartesian coordinate system buried the understanding of conics.

5. Time: 20:26–20:31

### **AN APPROACH TO REDUCE THE NUMBER OF FAILURE STUDENTS IN A LARGE CALCULUS CLASS**

Jianhui PAN

Chongqing University of Posts and Telecommunications, Chongqing, China

It is hard to teach students according to their aptitude for a large class and therefore a large class has a greater number of failure students. For a Calculus course the failure students usually lack the ability of solving math problems. To reduce the number of failure students for a large Calculus class, a new flipped instructing approach for problem solving is designed. The approach aims to enhance students problem-solving ability in term of high-quality teaching videos. The key idea of the approach is the flipped instruction. The flipped instruction not only enables students self-learning and sufficient practice, but also allows instructors to pay attention to failure students and help them to be successful as early as possible. The preliminary practice shows that this approach has a significant impact to reduce the number of failure students in a large Calculus class and can enhance the teaching and learning quality.

6. Time: 20:34–20:39

### **The Exponential Function from the viewpoint of Mathematical Modelling: a Chilean Lesson Study**

**Carlos Andres Ledezma Araya**, Elizabeth Montoya Delgadillo

University of Barcelona, Chile

It is intended to study the competence in mathematical modelling in tenth grade students (aged 15 to 16 years old) and thus characterise a working space and its complexities. A lesson plan was designed according to the Blomhøj and Jensen's didactic-cognitive model for the mathematical modelling process. The exponential function is considered as the main mathematical object into the lesson plan. The study was implemented in two phases: in the

first one, three interventions were carried out with tenth grade students, according to the Lesson Study methodology; in the second one, two binomials of tenth and eleventh grades students were studied respectively. The results analysed so far have evidenced the students achievements regarding the modelling process and the difficulties and mistakes in the written productions collected, also considering elements from the Mathematical Working Space Theory (MWS). This study is still under development.

7. Time: 20:42–20:47

### **USING OPEN EDUCATIONAL RESOURCES TO PROMOTE THE ACTIVE LEARNING OF CALCULUS IN URBAN DISTRICTS**

**Kenneth Horwitz**

New Jersey Institute of Technology, Cedar Grove, NJ, USA

In this study, I utilize the Open Educational Resource Active Calculus, for a summer program for high school students interested in STEM and professional development for in-service teachers called the Math Success Initiative. I will examine how the text facilitated utilizing the Inquiry-Based Learning (IBL) method to teach calculus. I will examine the essential elements of the Inquiry-Based method and take attendees through a particular lesson. I will also share some comments from students and in-service teachers about the sample lesson, IBL and their experience in this summer program.

8. Time: 20:50–20:55

### **MATHEMATICS ANXIETY LEVELS AMONG STUDENTS IN AN INQUIRY-BASED CALCULUS I CLASS**

**Harman Prasad Aryal, Otto Joshua Shaw**

Ohio University, Kapilvastu, Nepal

Mathematics anxiety is prevalent across the globe at all grade levels. It can impact an individuals everyday activities including numeracy and mathematics learning throughout their lives. This study investigated the mathematics anxiety among students who took an inquiry-based Calculus I course. The data were gathered from 71 students using the Abbreviated Mathematics Anxiety Rating Scale (AMARS) Likert type survey. Results include that 63% of students reported some level of mathematics anxiety and 80% indicated that they have mathematics test anxiety, whereas 72% indicated that they did not have numerical task anxiety and 55% indicated that they did not have mathematics course anxiety.

9. Time: 20:55–21:00

### **LEARNING DIFFICULTIES IN CALCULUS: AN INVESTIGATION THROUGH STUDENTS' WRITTEN SOLUTIONS**

**Raquel Carneiro Dorr**

University of Brasilia, Brasilia, Brazil

This paper aims to present two students written solutions selected during a first course of Differential and Integral Calculus in order to point out and illustrate students difficulties regarding basic mathematical contents necessary in algebraic procedures and to discuss the reasons as to why they not did not yet overcome these issues. The text ends with a discussion about the necessity of promoting, within the Brazilian mathematics education community, a willingness to formulate and develop methodological alternatives to handle the problem of learning difficulties from university beginners.

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### **Session 3, 17 July**

1. Time: 21:30–21:45

#### **The Design and Use of Low Instructional Overhead Tasks in Undergraduate Calculus: Making Student Reasoning More Accessible to Calculus Instructors**

**David C. Webb**

University of Colorado Boulder, Boulder, CO, USA

In undergraduate calculus classrooms designed to promote active learning, students are encouraged to offer conjectures, communicate their reasoning, and justify the validity of their arguments in the process of solving mathematics problems. The motivation for this approach is evidence from research that demonstrates how active learning can result in increased student persistence in the calculus sequence and improved students dispositions towards mathematics. This paper focuses on design principles for tasks that require less planning time than a traditional lecture but designed in such a way to increase active learning in the undergraduate calculus sequence i.e., low instructional overhead tasks. These principles for task design include use of multiple representations and organization of related prompts around key concepts to generate student ideas that can inform instruction. Using examples from a variety of instructional activities for calculus, this paper will highlight how to use, adapt and design calculus activities that are more conducive to instructor promotion of student interaction, reasoning and problem solving.

2. Time: 21:50–22:05

#### **THE OBSERVED IMPACT IMPLEMENTING INQUIRY-BASED LEARNING AT A CALCULUS CLASSROOM**

**Su Liang**

University of Texas–San Antonio, San Antonio, TX, USA

The data collected from a Calculus II class reveals that the structure of an inquiry-based learning (IBL) class produced rigorous on-going formative assessment during class teaching from three perspectives: helping the instructor see student thinking; helping the instructor see the level of student understanding; helping the instructor catch teachable moments. The rigorous on-going formative assessment, in turn, helped change student classroom behaviors in terms of asking more questions, showing deep thinking, and gaining confidence.

3. Time: 22:10–22:15

### **TEACHING CALCULUS BASED ON COMPLEXITY THEORY OF TEACHING AND LEARNING**

Mehmet Turegun

Barry University, Miami, FL, USA

An obsession with control permeates several aspects of our society and dominates our educational system throughout our classrooms in the United States. As an alternative to this control driven orientation, some aspects of the notion of decentralized control in a classroom environment may be worth exploring. In this paper, a description of the influence of the complexity theory of teaching and learning, as outlined by Davis & Sumara (2007), on my classroom practices in a calculus class is presented. An exploration of some alternative ideas for a calculus classroom based on a post-modern perspective with foci on the concepts of connectedness, non-linearity, holarchy, and creating spaces for community of truth is discussed.

4. Time: 22:18–22:23

### **NOTIONS OF CONTINUITY OF PRE-SERVICE TEACHERS; REFLECTIONS FOR A PROBLEMATIZATION**

**Antonio Bonilla**, Ricardo Cantoral, Ricardo Cantoral

Centro de investigacion y de estudios avanzados , Ciudad de Mexico, Mexico

In this paper we present some reflections on the results obtained in a study carried out with teachers in pre-service with respect to the notion of continuity. In the study it is proposed to model a logarithmic spiral through paper folding and dynamic geometry, the analysis of the result shows evidence that in the teachers drawing and discussions there is a notion of continuity which seems to differ from the formal definition of textbooks. From the socioepistemological program and with the indications of these results we seek to make a problematization around continuity, this process is to study original mathematical works and then compare the results with current textbooks.

5. Time: 22:26–22:31

## **RESIGNIFICATION OF THE DERIVATIVE IN A SCHOOL SITUATION WITH A PERSPECTIVE OF AN EXCLUSION - INCLUSION DIALECTIC: FROM EMULATION OF THE CONCEPT TO AUTONOMY OF USES**

**Jose Luis Morales Reyes**, Francisco Cordero Osorio

Cinvestav, Ciudad de Mexico, Mexico

This paper discusses theoretical-methodological aspects of Socio-epistemological Theory for the design of a school situation that will be socialized among mathematics teachers at the higher level. The design consists of resignifying the derivative based on epistemologies of uses of peoples mathematical knowledge at school, at work or in a profession, and in our daily life. The resignification of mathematics is a knowledge that is problematized with the teacher's school mathematics. Socialization, with an exclusion-inclusion dialectical perspective, will provide data on the processes of valuing of the uses of the derivative among teachers. Where, with semi-structured interview methods, the unit of analysis will be defined: from the emulation of the derivative concept to the autonomy of arguments of approximation, variation and transformation.

6. Time: 22:34–22:39

## **COVARIATIONAL REASONING: AN AXIS IN THE CONSTRUCTION PROCESS OF THE DEFINITE INTEGRAL CONCEPT**

**Mihaly Andre Martinez Miraval**, Martha Leticia García Rodríguez

Universidad Peruana de Ciencias Aplicadas, Lima, Peru

The study aims is to analyze different tasks which are proposed to the students related with the definite integral, and to show covariational reasoning appears and its used in the development of the mentioned tasks. For this reason, a documental research was done about definite integral, focused on the mathematical education field, and special attention was paid in the kind of tasks that were used and published in articles from 2008 to 2018. The students responses on this tasks were analyzed using the covariation levels defined in the theoretical construct of Thompson and Carlson (2018). From this perspective, the levels named no coordination, chunky continuous covariation and smooth continuous covariation, own of this construct, were identified in the students responses. We conjecture that its feasible to analyze the research from this perspective, in order to show the covariational reasoning is present and can be considered as an axis in the construction process of the definite integral concept.

7. Time: 22:42–22:47

## **THE "OVERGENERALIZATION OF LINEARITY": DIFFICULTY, CONFLICT OR OBSTACLE?**

**Nicolas Lopez**, Gloria Ines Neira Sanabria

Universidad Nacional de Colombia, Bogota, Colombia

Its analyzed an episode of class of derivative, based on the studies about the "overgeneralization of linearity", in which a student asks if the logarithm of a quotient can be expressed as the quotient of logarithms, a situation resulting from the Overgeneralization of what experience do as "true" for linear functions to nonlinear functions. The professor responds with a sarcasm, without being aware of that question expresses an obstacle and that most of the teachers do not even notice that this overgeneralization is an enhancer of errors, conflicts, difficulties and obstacles , constructs these all categorized in the context of the Onto-Semiotic Approach.

8. Time: 22:50–22:55

## **RATE OF CHANGE: MEANINGS STUDENTS HAVE IN ACCORDANCE WITH CONTEXT**

**Dafna Elias**, Prof. Dreyfus, Dr. Kouropatov, Miss Noah-Sella

Tel Aviv University, Tel-Aviv, Israel

A central issue for mathematics educators is how to make fundamental ideas of calculus meaningful for students. With respect to the derivative, the notion of rate of change (RoC) entails a coordination of understandings such as quantity, variation and relative change. Understanding in what extra-mathematical situations it is possible to talk about RoC is based on the understanding of co-variation which is a way of thinking of two quantities changing simultaneously. In this study, we asked students and teachers to assess whether it makes sense to talk about RoC in given situations, in order to gain insight into the ways students think about function and RoC, and the role of extra-mathematical context in students thinking about RoC. Our preliminary findings include notions which are considered necessary for the notion of RoC: quantities that change, invoking time into time-less contexts, a cause-effect relationship and a benefit gained from the discussion of RoC in the specific situation.