TSG Agenda

TSG–11: **Teaching and learning of probability.**

Class: A

Session 1, Tuesday July 13th

1. Time: 14:30─14:33

   Welcome

   Egan Chernoff & **Ernesto Sánchez**

   University of Saskatchewan, Canada, Canada; Departamento de Matemática Educativa, Cinvestav-IPN, México.

2. Time: 14:33─14:53

   A consideration of alternative sample spaces used in coin-toss problems.

   Amy Renelle, Stephanie Budgett & Rhys Jones

   The Department of Statistics at the University of Auckland, New Zealand.

Abstract. This paper examines coin-toss comparison questions from a pilot study. Considering possible sample spaces employed by participants, this is a reflection on whether one sequence could be more likely depending on the interpretation of the question. To critique the choice of sequences and determine possible scenarios in which one sequence may be more likely than the other, three alternative sample spaces were explored. It was determined that different sample spaces can lead to one sequence being more likely to occur than the other. Further evaluation discusses whether alternative sample spaces may have been utilised by the participants in the study and, hence, the paper concludes with an advocacy to enquire deeper into participants' reasoning when investigating coin-toss questions.

3. Time 14:5415:09

   High-school students' probabilistic reasoning when working with random intervals.

   Sandra A. Martínez Pérez & **Ernesto Sánchez**

   Departamento de Matemática Educativa, Cinvestav-IPN, México.

Abstract. This work presents the characteristics and results of a design experiment whose research objective is to understand the role that introducing the concept of random intervals plays in high-school students' probabilistic reasoning. It also seeks to identify the characteristics of the students' reasoning and the difficulties when students face problems involving random intervals. The participants were 16 students aged 16–17 in fourth semester at high school. The experiment consisted of physical and computational simulations to estimate the probability that a random interval contains a given number from the frequential approach.

4. Time 15:10–15:25

   Children's spatial cognitive strategies and their development from the perspective of microgenesis.

   Zikun Gong & **Du Zhang**

   Hangzhou Normal University, China; Xianlin Middle School, China

Abstract. This study chose 35 children aged 11 as the subjects and divided the Sample Space Tasks (SST) into three sections by considering their difficulty level. Taking ball-box model as examples, the model 1 is a simple task (take out one ball), the model 2 is a complex one (touching two balls), while the model 3 focused on independent repeated probability issues (touching a ball and put it back and touching the second ball). We explored students' cognitive strategy by conducting four rounds of investigations and experiments, that is pre-test, intervention, post-test and micro-occurrence method. The results shown that the main strategies for 11-year-old children are: based on accurate sample space (labeling method, connecting method and drawing tree diagram). The development of children's cognitive strategy shown that it develops rapidly from the second level to the third level. The development speed of strategic level is fast first and then slow. Children have the potential of learning sample space and can reach a higher level through teaching.

5. Time 15:26 – 15:41

   Teachers' epistemological assumptions that tend to govern their pedagogy while teaching probability.

   Haneet Gandhi

   Department of Education, University of Delhi. INDIA

Abstract. This study addresses the epistemological assumptions that seem to govern teachers' pedagogic strategies when they use random generators for teaching probability concepts, and how teachers make transitions between the three approaches of probability: namely, classical, frequentist and propensity approaches. Following the tenets of the interpretative paradigm, design tasks have been used in this study to know 42 teachers' prevalent epistemic considerations as they worked with an isomorphic resource. Sequencing teachers' pedagogic strategies related to an unfamiliar object (in teaching probability), helped in unravelling their dispositions to the perceptual properties, structural aspects of the object and other external generating conditions as determinants for their probabilistic deductions.

6. Time: 15:42 – 15:57

   The emerging interplay between subjective and objective notions of probability in young children

   Sibel Kazak & Aisling Leavy

   Pamukkale University, Turkey; University of Limerick, Ireland

Abstract. To promote the development of young children's probabilistic intuitions through experience, we focus on the dual nature of probability, which is both subjective from an epistemological perspective and objective from a statistical perspective. This paper reports on a task design to bridge the two notions of probability and the task-based interviews with 7–8-year-olds. We explore how these two conceptions of probability interact in children's estimations of the likelihood of outcomes from chance events as new information is obtained from the data collected through both physical experiment and computer simulation. Our findings suggest that even though children have not reached the level of quantitative reasoning yet, several of them were able to use the data in a relatively reasonable way to update their subjective probabilities based on their personal beliefs.

7. Time 15:58–16:08

   Establishing connections between language and probabilistic notions through a WODB task.

   Maria Ricart, Pablo Beltrán-Pellicer & Assumpta Estrada

   University of Lleida, Spain; University of Zaragoza, Spain

Abstract. In this study, we analyse linguistic elements used by students of different educational levels (Early Childhood Education, Primary Education, Secondary Education and Higher Education) when they discuss around a WODB activity about probability, in order to explore their probabilistic ideas. The results allow to identify how and when students acquire the notion of equiprobable events and impossible event. It is concluded that this task can help teachers understand both cognition and language about probability of their students and, consequently, improve the implementation of mathematical instruction processes.

8. Time 16:09–16:19

    Problem sequences for developing two basic notions: probability and expected value in Hungarian secondary schools.

    Oedoen Vancso & Eszter Varga

    Eötvös Loránd University, Budapest; Bornemissza Péter Highschool, Budapest

Abstract. This paper deals with a possible series of problems, with the today popular online betting in its focus. The goal is to foster students understanding and usage of such fundamental notions their as probability and expected value. The traditions of Hungarian Mathematics Education will be introduced and its connections with international trends will be demonstrated. After a summary and description of historical and theoretical background we will briefly introduce the core problem and some preparing and discussion problems.

9. 16:20 – 16:30

    Understanding elements of a randomization test

    Susanne Podworny

    Paderborn University, Germany

Abstract. Drawing inferences from data is a skill that is often needed in everyday life. Almost every day we encounter situations in which decisions have to be made based on data. Randomization tests are an approach to introduce learners to inferential reasoning, but several elements are needed to conduct a randomization test. Which elements do learners can cope with and which ones they struggle with is presented in this paper.

Session 2, Wednesday July 14th

10. Time 19:30 – 19:49

Self-reported practices of probability teaching: the use of the frequentist approach, manipulatives and technological tools

Vincent Martin, Mathieu Thibault & Marianne Homier

Université de Sherbrooke, Québec, CA, Université du Québec en Outaouais, Québec, CA

Abstract. The first stage of a study in Québec enabled us to draw up a statistical portrait of probability teaching practices self-reported by 626 teachers at the primary and secondary levels (with students aged from 6 to 17 years old). For the second stage of the study, discussed here, we wanted to elaborate on some of the questionnaire answers and to discuss professional development avenues inspired by the teachers' experience. We conducted one-hour individual interviews with eight teachers at various levels of primary and secondary school who had taken part in the first stage and whose self-reported teaching practices were considered to be exemplary according to five criteria. By means of a content analysis of their discourse, we explore issues surrounding certain self-reported probability teaching practices through examples related to the use of the frequentist approach, manipulatives and technological tools. Firstly, for the use of the frequentist approach in probability teaching, the way the situations are carried out varies from one participant to another, in terms of the person running the trials (students or teacher), the size of the generated samples and the compilation method for the trials that were done. Secondly, regarding the use of manipulatives in probability teaching, the participants mentioned using unusual manipulatives for different reasons: to enable students to visualize a situation, to run trials or to support students' probabilistic reasoning. Thirdly, for the use of technological tools in probability teaching, the participants mentioned using a simulator, but some mentioned the use of other technological tools, including spreadsheets, programming tools, videos, and websites. Furthermore, if the intentions behind the use of these technological tools vary, the participants all stated that they used these tools themselves most of the time.

11. Time 19:50 – 20:05

Is it in the cards?!? Revealing consequential probability

Egan Chernoff[1], Nat Banting[2] & Ryan Banow[1]

[1]University of Saskatchewan, Saskatchewan, Canada. [2]Marion M. Graham Collegiate. Saskatchewan, Canada.

Abstract. A number of memorable tasks have been revealed through collegial exchanges with underlying philosophical, theoretical, potentially nefarious motivations. Such was the origin of the probability problem, and the mutual difference of opinion presented here. This article recounts how, together, we explore and quarrel about the probabilities stemming from a simple standard deck of cards sitting on a table, with moments of consensus spiraling into low points of discontent and back again, as our actions and reactions create intriguing, yet disconcerting, consequences. Opposing arguments are presented in hopes of arriving at consensus, but maybe, just maybe, it is not in the cards.

12. Time 20:06–20:16

The frequentist approach of probability, from random experiment to sampling fluctuation

Jannick Trunkenwald[1], Fernand Malonga Moungabio[2] & Dominique Laval[1]

[1]LDAR Université Diderot Paris 7, France : [2]Université Marien Ngouabi, Congo

Abstract. This article deals with an introduction of probability in high secondary school through sensory world. This approach is based on sampling fluctuation with empirical observations of frequencies. Initially, we analyze how students work on such a task by using dices, pencil and paper. We then identify the use of schemes and data visualization by students. Considering scheme as a primitive form of algorithm, we observe interactions between algorithmics and mathematics. In class, students know how to use notion of frequency, but linking frequencies with probability appears as a difficult issue. This enables us to identify entry points, as well as sticking points that could be avoided by the uses of computing. We then decide to consider contribution of algorithmics to introduce frequentist approach.

13. Time 20:17 – 20:27

Alice in Randomland: differences in attitudes of future primary school teachers towards probability and its teaching

Claudia Vázquez, **Flavio Guiñez**, Camila Brito & Salomé Martínez

Universidad de Chile and Pontificia Universidad Católica (Chile)

Abstract. This paper reports the results of a study that seeks to investigate how attitudes towards probability and its teaching of future primary school teachers are influenced by interacting with the book "Alice in Randomland" aimed to stimulate the interest in probability and to provide opportunities for its learning. For this, a scale of attitudes towards probability and its teaching was applied to 40 future teachers, before and after interacting with this didactic resource. Preliminary results show significant changes in student attitudes, particularly in the affective component towards probability.

14. Time 20:28 – 20:38

Modeling eliciting activities for the teaching of the probability in a computer environment.

Santiago Inzunza

Universidad Autónoma de Sinaloa, Culiacán, MX

Abstract. In this article we describe the results of a learning trajectory based on Model Eliciting Activities (MEA) in a computer environment for an introductory probability course with university students. The results show that model building was a difficult activity for students in the initial phase of the learning trajectory. Among the main difficulties is the lack of identification of key components of the problem and consider equiprobable events that are not (equiprobability bias). In the problems that frequency information is available as a starting point, the students built the model with the observed data, without taking into account the frequency variability. The students were aware of the relevant information for modeling as the trajectory activities were developed.

15. Time 20:39 – 20:49

Developing a learning progression for probability based on the GDINA model in China.

Bai Sheng-nan & Han Ji-wei

Collaborative Innovation Center of Assessment toward Basic Education Quality, Beijing Normal University, Beijing, 100875; School of Mathematics and Statistics, Northeast Normal University, Changchun, Jilin, 130024.

Abstract. This research focuses on developing a learning progression of probability for middle school students, and it applies GDINA model in cognitive diagnosis models to data analysis. GDINA model analysis firstly extracted nine cognitive attributes and constructed their attribute hierarchy and the hypothesized learning progression according to previous studies, curriculum standards and textbooks. Then the cognitive diagnostic test was developed based on Q-matrix theory. Finally, we used GDINA model to analyze a sample of 1624 Chinese middle school students' item response patterns to identify their attribute master patterns, verify and modify the hypothesized learning progression. The results show that, firstly, the psychometric quality of measurement instrument is good. Secondly, the hypothesized learning progression is basically reasonable and modified according to attribute mastery probability. The results also show that the level of probabilistic thinking of middle school students is improving steadily. However, students in grade 8 are slightly regressive. These results demonstrate the feasibility and superiority of using cognitive diagnosis models to develop a learning progression.

16. Time 20:50–21:00

Secondary school students' strategies in solving permutation problems.

Lamanna Luca, Magdalena Gea & Carmen Batanero

Free University of Bozen-Bolzano, Italy; University of Granada, Spain

Abstract. This study is part of a wider investigation which aims to explore the effect of instruction on the combinatorial reasoning of secondary school students in Italy, focusing on students' performances and on strategies developed during problem solving activities. Two groups of students – one with instruction (51) and one without (64) – were given a questionnaire consisting of 13 problems solvable through the application of different combinatorial operations. This study focuses on the results obtained in the two items of the

questionnaire dealing with simple permutations. All the questionnaires were analysed, and the strategies used by the students were described, also dividing the results according to each of the variables considered. Even though results show a persisting lack in students' combinatorial abilities across both groups, an overall increment in the number of correct answers was observed in the smaller group of students. This suggests that instruction has, in general, a positive effect on students' reasoning. The two groups demonstrated a different development of the strategies; students who did not received explicit instruction mainly developed constructive strategies such as enumeration – used both as a primary strategy and as a resource to support other strategies – while the students of the second group tended to prefer arithmetic strategies or formulas. Instruction seems to have an effect on students' combinatorial reasoning, in agreement with the findings of Fischbein and Gazit (1988), even though many students with instruction still use enumerative strategies. The study provides a first exploration aiming towards the characterization of the evolution secondary school students' combinatorial reasoning, even though results were obtained from a relatively small sample, remarking how instruction should be considered as an active variable in the development of combinatorial reasoning.

Session 3, Saturday 17th

17. Time: 21:30 – 21:50

From towers of linking cubes to the binomial expansion theorem: what can be learned about combinatorics?

Gale Russell

University of Regina

Abstract. This paper presents an account of the ongoing development of a multi-tiered task that I have been using with third year university pre-service teachers with a variety of mathematics backgrounds (one university mathematics class, because they had to take it, to math majors) and mathematical confidence levels (spanning from expert to terrified). I have also had opportunities to do these tasks with all levels of mathematics teachers and mathematicians, and the task has started to be used within middle years and high school mathematics classes with positive anecdotal results.

The task starts with the posing of the following question: "How many different 3-cube high towers can be built using only two colours of linking cubes," a question I first asked to gather evidence of my pre-service teacher students' understanding of permutations and

combinations. My paper speaks to how that initial question was engaged with, how I have continued to add more questions to follow the first, and how those questions have taken all of us on a learning journey through permutations and combinations, language in mathematics learning and communicating, abstraction and generalization from patterns, providing arguments in defense of conclusions, Pascal's triangle, and the Binomial Expansion Theorem. Rather than a paper that shares quantitative data and analyses, this paper is meant to be a sharing of ideas, ponderings, and possibilities in the hopes of further learning and collaborations.

18. Time 21:51 – 22:06

How can probability reasoning protect adolescents from problem gambling?

Caterina Primi & Maria Anna Donati

NEUROFARBA - University of Florence. Italy

Abstract. Despite the restrictions to gamble for youth, prevalence studies report that a large number of adolescents are involved in gambling activities and that they are at higher risk for developing gambling problems compared to adults. Some cross-sectional studies revealed that adolescent problem gamblers are more prone to mistaken views about randomness when compared with non-problem gamblers, and they held erroneous beliefs about their chance of winning. The aim of this study was to develop and evaluate a school-based preventive intervention aimed to modify gambling-related distortions on at-risk adolescents, focusing the training activity on the concept of probability. Participants were 72 adolescents (89% males, mean age=16.87, SD=1.14 years) attending technical high school in a neighborhood of Florence (Italy). An experimental design was conducted with two groups (Training vs. No Training), and two measurement times (pre-test and post-test sessions). Participants were randomly assigned to each group. After 1 week from the pre-test, the intervention began with the first didactic unit which was aimed at introducing the concept of probability, in particular the independence of random events and equiprobable and non-equiprobable random events. The intervention was attended only by the Training group. After 1 week, there was the second didactic unit, which was focused on reasoning in probabilistic terms inside cold and hot contexts. After another week, the post-test was administered to both the groups. Results showed a significant interaction between groups and time (F $(1,54)$ = 8.14, p = .006, $\eta^2 p$ = .170). Adolescents who attended the intervention reported a significant reduction of gambling-related distortions. Findings suggest that teaching probability is effective in

reducing the susceptibility to gambling-related distortions by acting on the reasoning competencies useful to reduce the susceptibility to erroneous cognitions about gambling with regards to personal control on gambling outcomes.

19. Time 22:07–22:22

The Mathematical work of secondary teachers in the domain of probability in Chili.

Katherine Machuca Pérez

Pontificia Universidad Católica de Valparaíso

Abstract. The purpose of this qualitative research is to study the mathematic activity in front of tasks resolution into the domain of probability context of mathematic modelling, which is shown by the high school mathematics teacher in pre-service and in-service training. We consider the Mathematical Working Space (MWS) model as a theoretical support, methodological design, case study of a team of 27 teachers. We will analyze mathematical objects under a semiotic, instrumental and discursive dimension, identifying vertical and privileged plains, the paradigm applied and the different strategies used. We will show our results by contrasting "a priori" and "a posteriori" analysis about the implemented task.

20. Time: 22:23 – 22:33

The computer simulation as a resource to teach normal distribution.

Jesús Salinas & Julio César Valdez

Colegio de Ciencias y Humanidades, Plantel Vallejo, UNAM, México.

Abstract. This manuscript shows the preliminary results of a teaching experiment with which we seek to analyze the potential that technology has as a resource to teach Normal distribution. To do so, we designed a lesson whose main approach was the use of an educational software. The lesson was applied to a group of high-school students in a regular course. Among the main results, we observed that the computer simulation allowed the subjects to use the notion of normal distribution for the approximation of the solution to a binomial problem. Then, the use of the software, under the teacher's guidance, helps the students' learning due to its intuitive approach to concepts that are difficult to understand formally, such as the Law of Large Numbers and the Central Limit Theorem, which represent a key support to study normal distribution.

21. Time 22:34 – 22:44

Use of the empirical rule in the course of probability: an application proposed by students.

Beatriz A. Rodríguez González, Gabriela Noemí Figueroa Ibarra, Omar Alejandro Guirette Barbosa, Héctor Antonio Durán Muñoz & Difariney González Gómez

Universidad Politécnica de Zacatecas, México; Universidad Nacional de Colombia, Colombia.

Abstract. The interest of this paper lies in the current importance in the field of research teaching of statistics, which have been highlighted major difficulties for students to understand the concepts related to probability. In this regard, we have chosen the concept of empirical rule and practical application created by 30 students (under the approach of problem posing) use, in order for students to engage some measures of central tendency and dispersion of descriptive statistics, so students can calculate and understand the use of the rule using the empirical probability approach.

22. Time 22:45-22:55

Confidence and competence of Indonesian secondary school students in completing probability tasks.

Bustang Bustang

Loughborough University and Universitas Negeri Makassar. England, UK.

Abstract. This study investigated secondary school students' confidence and competence in solving probability problems relating to three well-known probability misconceptions, namely: representativeness heuristic, equiprobability bias, and ratio bias. An 11 multiple-choice-item questionnaire, inviting requests for students' justifications and confidence ratings, was given to 160 Indonesian secondary school students (aged 14-15). Following this, several students were interviewed based on their responses to the questionnaire, representing the variety of response and justification types for each of the items in the questionnaire. Students' responses and reasoning, along with their confidence ratings, are discussed in order to reveal Indonesian secondary school students' confidence and competence in responding to various probability problems.

23. Time 22:56–23:00

Closure

Egan Chernoff & Ernesto Sánchez

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