

TSG Agenda

TSG 7: **Teaching and Learning of Algebra at Secondary Level** (number and title)

Class: **A** (Class A for TSGs with odd numbers; Class B for TSGs with even numbers)

****Please prioritize the sessions in “core-time” (from 19:30-23:00, Beijing time, i.e. Session 2, 3 for Class A and session 1, 2 for Class B) as they are friendly to most of the time zones in the world.**

Session 1 (13th July 2021, 14:30 – 16:30, Beijing time)

1. Time: 14:30 – 14:40

Welcome and housekeeping matters by Chair of TSG7

Session Chair then takes over.

(Session Chair: Dr. Levi Elipane)

2. Time: 14:40 – 15:00

Title of the Paper:

Knowledge for teaching algebra: variation in the use of knowledge in the light of classroom constraints

Author(s) (with the presenter name in BOLD if more than 1 name here):

Demonty Isabelle¹, Vlassis Joëlle²

Institution(s) (to school/department/research center) and Country/Region:

1. University of Liège/Department of psychology, Speech Therapy and Education Sciences/ Research Unit EQUALE (Evaluation and quality of teaching), Belgium

2. University of Luxemburg/Faculty of Humanities, Education and Social Sciences/Department of Education and Social Work, Luxembourg

Short abstract of the paper (20 lines maximum):

This study illuminates knowledge for teaching elementary algebra by studying the variation in teachers' knowledge and by interpreting it with reference to the classroom practices of particular relevance to algebra lessons. We examine this variation by analysing the results of a paper-and-pencil questionnaire submitted to 88 middle school teachers from Luxembourg. The results show that variations in the use of knowledge are considerable and that the interpretation of this variation in terms of classroom practices helps to understand important factors inherent to algebra teaching that foster or hinder teachers' use of their knowledge in some situations: thus, when required to stand back from their own fluency in the use of algebra in order to anticipate the approaches that will be used by students exploring the subject for the first time, the teachers show an awareness that students may use arithmetical approaches or that they may find it difficult to assign meaning to the letters involved in algebraic calculations. On the other hand, when this same knowledge is involved in situations where they deem it essential to reduce the complexity of tasks in order to make them accessible to students, they tend to give weight to the belief that algebraic reasoning is beyond the reach of students starting out in this area. In this sense, the goals that teachers assign to their practices have a significant impact on the actual deployment of their knowledge.

3. Time: 15:00 –15:20

Title of the Paper:

Constructing the link between graphical visualization and algebraic computation by means of analogy: the case of a system of equations.

Author(s) (with the presenter name in BOLD if more than 1 name here)

Klila Copperman¹, Anatoli Kouropatov²

Institution(s) (to school/department/research center) and Country/Region

1. Jerusalem College of Technology, Israel
2. Levinsky College of Education, Israel

Short abstract of the paper (20 lines maximum):

This paper reports a part of a study conducted in an attempt to clarify students' knowledge construction process in the case of solving problems that require the use of different representations of a system of equations. Following Fischbein (1987) we consider graphical and algebraic-symbolic representations of a system of equations as an intra-mathematical analogy, in which the origin is a symbolic expression and the analogue is more intuitive, usually a geometric representation. We conjecture that the emergence of that analogy in students' minds could be helpful in constructing the link between the representations.

For analyzing the students' knowledge construction process, we adopted the theory of Abstraction in Context, which is recognized as a suitable theoretical framework and provides appropriate methodological tools (Dreyfus, Hershkowitz & Schwarz, 2015).

The study included 12 pairs of students; eight pairs were secondary school students, half of them learned mathematics at a regular level, and half learned mathematics at an advanced level, another four pairs of students were first year engineering students. The study was conducted as a two-hour clinical interview in laboratory conditions. The students' work was documented and transcribed. Our findings suggest three types of links between the algebraic and graphical representations that were constructed in the process of working on the problem. We argue that our findings could be interpreted as evidence for knowledge construction that involves the emergence of the analogy.

4. Time: 15:20 –15:30

Title of the Paper:

Using an online card game-based activity to build algebra foundation

Author(s) (with the presenter name in BOLD if more than 1 name here)

Jiqing Sun

Institution(s) (to school/department/research center) and Country/Region

Deakin University, Australia

Short abstract of the paper (20 lines maximum):

The transition between arithmetic and algebraic is challenging for lower-secondary students. Two notable challenges for students' algebraic thinking are the understanding of pronomeral, and results-orientated thinking from arithmetic (also known as 'computational thinking'). Many researchers are contributing to exploring instructional approaches in addressing these issues. Nowadays, digital technology should play a prominent role in this field, and its affordance still needs to be further investigated. To respond to this call, this report illustrates how an online card game based activity supports students' understanding of pronomeral, and the departure from computational thinking in an everyday classroom context.

5. Time: 15:30 –15:40

Title of the Paper:

Investigating students' algebraic proficiency from a symbol sense perspective

Author(s) (with the presenter name in BOLD if more than 1 name here)

Al Jupri

Institution(s) (to school/department/research center) and Country/Region

Department of Mathematics Education, Universitas Pendidikan Indonesia, Indonesia

Short abstract of the paper (20 lines maximum):

Symbol sense refers largely to an ability to give meaning and to see important structures to symbols, expressions and formulas. This ability shows student algebraic proficiency in dealing with symbolic representations and is considered important for students of prospective mathematics teachers. This study, therefore, aims to investigate students' algebraic proficiency in terms of symbol sense. To do so, we set up an exploratory study. First, after conducting a literature study, we designed symbol sense tasks according to symbol sense characteristics on the topic of equations. Second, we administered an individual written test involving 31 students of prospective mathematics teachers (18-19 year-old) at the beginning of School Mathematics course. Third, we analyzed written student work using a symbol sense perspective. The results showed that about two-thirds of the participated students lack of symbol sense in which they tend to use more procedural strategies than symbol sense strategies in solving equations.

6. Time: 15:40 –15:50

Title of the Paper:

Diagnosis and treatment of students' algebraic misconceptions and errors

Author(s) (with the presenter name in BOLD if more than 1 name here)

Mukunda Prakash Kshetree

Institution(s) (to school/department/research center) and Country/Region

Tribhuvan University, M. R. Campus (Dept of Maths Ed), Kathmandu, Nepal

Short abstract of the paper (20 lines maximum):

This study has demonstrated the exploration and diagnosis of students' common patterns of algebraic misconceptions and errors (M/Es), and their treatment effect. The guiding philosophy of the study was Fallibilism and adopted theory was Constructivism. The treatment was prepared by developing special teaching episodes (based on students' M/Es) and implemented them through cooperative learning approach (CLA) with remedial strategies. The theories applied for the remedial strategies were Behaviorism, Cognitivism, Social learning and Conceptual change.

The study had adopted a mixed methods research followed by quasi-experimental design with non-equivalent control groups in order to have also a comparative study between control and experimental groups of the students. The qualitative information were mainly collected through in-depth interviews of the students which were supplemented by minute observation of peer groups' activities in the classrooms, making notes, and administering two sets of questionnaires among them. Similarly, the sources of quantitative data were the scores obtained by the students of both the groups in pre-and-post tests.

The study has revealed numerous types of M/Es under seven categories with some of their prototypes. Most of the M/Es were found related to lack of conceptual understandings, deficient procedural knowledge/skills and implicational inabilities. Moreover, some of the non-cognitive causes of M/Es explored were as guessing, hitting and missing, weak arithmetical skills, low confidence, anxiety, nervousness, and carelessness. Further, the intervened treatment was found most effective to treat students' M/Es. As a result, the students of experimental groups made a significant progress compared to those of control groups.

7. Time: 15:50 –16:00

Title of the Paper:

Examining the quality of classroom interactions in the teaching of Algebra for upper secondary schools

Author(s) (with the presenter name in BOLD if more than 1 name here)

Aline Dorimana¹, Alphonse Uworwabayeho² and Gabriel Nizeyimana²

Institution(s) (to school/department/research center) and Country/Region

1. University of Rwanda College of Education/ African Center of Excellence for Innovative Teaching and Learning Mathematics and Science (ACEITLMS), Rwanda

2. University of Rwanda College of Education, Rwanda

Short abstract of the paper (20 lines maximum):

Teaching for conceptual understanding or problem-solving that promote reasoning has become increasingly important for learners in the discussion of learning mathematics effectively. Such teaching needs to be grounded in the rich classroom interactions through lenses of student engagement and teacher facilitation on the learned content. The present paper continues to support these by examining the quality of classroom interaction to reinforce the teaching for conceptual understanding at one school in Kayonza district, Rwanda. The Mathematical Classroom Observation Protocol for Practices (MCOPP) was used to capture the behavior of two teachers and 82 learners of 2 grade 11 classes (learners' age, 15-19 years) involved in the study. 14 episodes of algebra lessons were studied. The findings of the study indicate that participating teachers present limited aptitude for the creation of interactive classrooms environment. Therefore, there is a need to support teachers to create an environment that allows them to scaffold learners learning in the long term to develop conceptual understanding.

8. Time: 16:00 –16:20

Q & A for Short Oral

9. Time: 16:20 –16:30

Sum up and closure of Session 1

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Session 2 (14th July 2021, 19:30 – 21:00, Beijing Time)

1. Time: 19:30 – 19:40

Housekeeping matters by Session Chair (Dr. Yali Pang)

2. Time: 19:40 – 20:00

Title of the Paper:

Generalization as a marker for robust mathematical meanings among in-service algebra teachers

Author(s) (with the presenter name in BOLD if more than 1 name here):

Lori Burch, Erik Tillema

Institution(s) (to school/department/research center) and Country/Region:

Indiana University Bloomington, United States of America

Short abstract of the paper (20 lines maximum):

We report on a teaching experiment with two in-service algebra teachers. The purpose of the study was to identify how their initial mathematical meanings (MM) for advanced algebraic identities developed over the course of the experiment. The experiment used combinatorial problem situations to accomplish this goal. The purpose of this paper is to outline how generalizing actions can be used as a marker for increasingly robust MM. The primary contribution of the paper is theoretical in that the intent is to conjoin two lines of research—research on MM and research on generalization. We consider this connection important because generalization is central to algebraic reasoning.

3. Time: 20:00 – 20:20

Title of the Paper:

Student knowledge of exponential functions

Author(s) (with the presenter name in BOLD if more than 1 name here):

Robert Powers¹, Alees Lee¹, Melissa Troudt², & Jodie Novak¹

Institution(s) (to school/department/research center) and Country/Region:

1. University of Northern Colorado, United States of America
2. University of Wisconsin – Eau Claire, United States of America

Short abstract of the paper (20 lines maximum):

This study investigated responses on a high school assessment of exponential growth as multiplicative in nature and in contrast to linear growth. We developed tasks based on learning progressions and curricular objectives of the Common Core State Standards in Mathematics. We administered the assessment in four high school classrooms as part of a larger study. Analysis of student responses distinguished student knowledge and skills as essentially correct, partially correct, and incorrect.

4. Time: 20:20 – 20:30

Title of the Paper:

The importance of teacher-student interactions in mathematical learning: the example of generalisation

Author(s) (with the presenter name in BOLD if more than 1 name here):

Vlassis Joëlle¹ & Demonty Isabelle²

Institution(s) (to school/department/research center) and Country/Region:

1. University of Luxemburg, Luxemburg, Faculty of Humanities, Education and Social Sciences, Department of Education and Social Work, Luxemburg.
2. University of Liège, Department of psychology, Speech Therapy and Education Sciences, Research Unit EQUALE (Evaluation and quality of teaching) Belgium

Short abstract of the paper (20 lines maximum):

This paper considers the importance of teacher-student interactions in the learning process. It analyses these interactions in the context of a generalisation activity in algebra at the start of secondary education. In this paper, we first recall Radford's (2008) definition of learning as a social process mediated by signs aimed at producing conceptual objects. On this basis, we show the unavoidable need for quality interventions on the part of the teacher in exchanges with students. In the field of generalisation, authors usually point out that the potential to develop a rich and meaningful discourse is especially likely to arise when the

teacher encourages students to justify and clarify their statements or draws attention to the mathematical relations of a problem, and so on (Ellis, 2011). However, for Callejo & Zapatera (2017) this is not sufficient. These authors raise the idea of basing these interactions on ‘professional noticing’ – a form of observation dependent on the ability to both identify significant mathematical elements and to interpret students’ mathematical comprehension in a given situation. To this end, they draw attention to one particular aspect of professional noticing, namely students’ mathematical thinking; they argue that this means more than just taking students’ ideas into account. Finally, based on an excerpt from a teacher-student interaction, we highlight the difficulty for the teacher of interacting appropriately by pinpointing students’ mathematical thinking. This can indeed represent quite a challenge for teachers especially in the heat of classroom action.

5. Time: 20:30 – 20:40

Title of the Paper:

Learners’ number patterns generalisations in a South African evaluative assessment

Author(s) (with the presenter name in BOLD if more than 1 name here):

Zwelithini Dhlamini

Institution(s) (to school/department/research center) and Country/Region:

Department of Mathematics Science and Technology Education, South Africa

Short abstract of the paper (20 lines maximum):

This paper presents an analysis of South African learners’ generalisation of number patterns in an Annual National Assessment. Content analysis was employed as method and design to tap into $n = 1250$ learners’ responses using the Generalisation Strategy Framework lens. The results indicate that learners inappropriately generalized number patterns due to improper use of their functional thinking related to chunking, and recursive strategies. The learners’ failure of unitizing, the realization that the number pattern was a cubic, resulted in the formulation of irrelevant generic rules of the generalisation of number patterns. The suggestion is that instruction should focus on making learners conscious of the relevant strategy to employ for their generalisation of number patterns.

6. Time: 20:40 – 20:50

Q & A for Short Oral

7. Time: 20:50 – 21:00

Sum up and closure of Session 1

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Session 3 (17th July 2021, 21:30 – 23:00, Beijing Time)

1. Time: 21:30 – 21:40

Housekeeping matters by Session Chair, Dr. Michael Steele

Introduction of Invited Speaker by Session Chair

2. Time: 21:40 – 22:10

Title of Invited Talk:

Thinking about Algebra from the Anthropological Theory of the didactic: Reference models for the analysis and the design

Author(s) (with the presenter name in BOLD if more than 1 name here):

Noemí Ruiz-Munzón¹, Marianna Bosch², Josep Gascón³

Institution(s) (to school/department/research center) and Country/Region:

1. Universitat Pompeu Fabra-Tecnocampus, Spain
2. Universitat Ramon Llull, Spain
3. Universitat Autònoma de Barcelona, Spain

Short abstract of the paper (20 lines maximum):

Questioning the knowledge to be taught appears as a critical question in mathematics education research. The case of school algebra is not an exception. What mathematical activities are proposed at school as algebraic tasks? What others fail to appear as such, and why? What roles and connections with other contents does algebra have at school? Which ones fail to appear and why?

The anthropological theory of the didactic (ATD) proposes to approach this questioning by assuming that school algebra is an institutional construction that responds to certain societal needs about education, that inherits historical constraints related to old educational and social organizations, and that researchers need to revisit regularly. Researchers' conceptions about algebra are formulated in terms of reference epistemological models, which are used for the analysis and design of teaching and learning processes. Making researchers' epistemological models explicit facilitates its discussion, confrontation and development (Lucas, Fonseca, Gascón, & Schneider, 2020).

3. Time: 22:10 – 22:20

Title of the Paper:

Students' unconventional graphical representations of covariational reasoning

Author(s) (with the presenter name in BOLD if more than 1 name here):

Laurie Cavey, Tatia Totorica, Patrick Lowenthal

Institution(s) (to school/department/research center) and Country/Region:

Boise State University, United States of America

Short abstract of the paper (20 lines maximum):

This study is situated within the context of a design-based research project with the goal of developing video-based learning modules to support teacher education candidates' ability to attend to student thinking. As part of the content development process for one module, we analyzed the graphical representations from more than 100 USA grades 6-12 students as they completed our version of the well-known bottle filling task. The instances of students' written work that were unexpected and initially difficult to interpret were of particular interest for their potential use in broadening candidates' ability to interpret students' graphical representations. Driven by a pedagogical lens that aspires to connect students' informal intuitive ideas to more formal mathematical representations, we investigated these instances and developed hypotheses about the productive ideas associated with two types of unconventional graphical representations. Implications for analyzing students' graphical representations of their covariational reasoning will be shared.

4. Time: 22:20 – 22:30

Title of the Paper:

The Impact of an Online Learning Platform in Algebra

Author(s) (with the presenter name in BOLD if more than 1 name here):

Zachary Stepp

Institution(s) (to school/department/research center) and Country/Region:

School of Teaching and Learning, College of Education, University of Florida, United States of America

Short abstract of the paper (20 lines maximum):

Algebra Nation is an online teaching and learning platform developed in 2012 for secondary students in the United States. It has been adopted by several states across the US and partnered with teachers, students, administrators, parents, schools, and communities to provide personalized, adaptive tools for mathematics learning. The platform includes multiple instructors, a personalized learning plan, a 24/7 discussion board, alignment with standards, and a virtual teacher professional development network. This synthesis of a growing body of research on Algebra Nation highlights some of the positive impacts it has had in schools and classrooms across the US. It also presents key findings related to what makes an effective instructional video. With a large pool of data over several years, future studies should look more longitudinally at the demographics.

5. Time: 22:30 – 22:40

Q & A for Short Oral

6. Time: 22:40 – 23:00

Sum up by Session Chair

Closure by TSG Chair

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Note:

Class A:

- Session 1: 14:30-16:30 Beijing time, July 13th
- Session 2: 19:30-21:00 Beijing time, July 14th
- Session 3: 21:30-23:00 Beijing time, July 17th

Class B:

- Session 1: 19:30-21:00 Beijing time, July 13th
- Session 2: 21:30-23:00 Beijing time, July 16th
- Session 3: 14:30-16:30 Beijing time, July 17th