ADVANCES IN ARGUMENTATION AND MATHEMATICS EDUCATION

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Abundant research has been done on non-deductive aspects of mathematics. In How to Solve It (1948) and Plausible Reasoning (1954), Pólya uncovered the heuristics that are useful for approaching mathematical problems. The most common state of mind of the problem solver is uncertainty. In Pólya's words, the problem solver engages in 'plausible reasoning'. In Proofs and Refutations, Lakatos (1978) went deeper into Pólya's idea of heuristics. He exemplified series of attempted proofs that mathematicians offered for conjectures, only to be repeatedly refuted by counter-examples. Pólya and Lakatos deeply influenced progressive pedagogies in mathematics. More recent accounts show that non-deductive reasoning processes are very frequent in their activity. Jaffe and Quinn (1993) point at the omnipresence of what they call 'casual reasoning' (that we call 'monological argumentation', which enables the formation of conjectures. Many mathematicians reacted to Jaffe and Quinn on the role of conjectures in mathematical activities, and stressed the centrality of understanding in mathematics and the role of proving in this endeavor. In our own words, argumentation is social/dialogical – in order to explain and convince (Thurston, 1994). A growing literature shows the centrality of monological and dialogical argumentation in elaborating and understanding mathematical ideas. For example, Weber, Inglis and Mejia-Ramos (2014) showed that many mathematicians sometimes gain high levels of conviction with empirical or authoritarian evidence and sometimes do not gain full conviction from the proofs they read. In another study Inglis, Meiji-Ramos and Simpson (2007) exemplified young mathematicians' argumentation using Toulmin's full diagram and showed that modal qualifiers (e.g. expressions of doubt or of confusion) play an important role. These results suggest that instruction should be less centred on fostering deductive processes.

Besides the recognition of the importance argumentation in mathematics and in mathematics education, considerable efforts have been invested out of the world of mathematics in theories of argumentation (e.g., structural and monological for Toulmin theory, or discursive and dialogical for van Eeemeren and Grootendorst theory) and on the contribution of argumentation to learning processes and outcomes (Schwarz & Baker, 2016). We will show how these advancements impinge in designing tasks in mathematics that afford argumentation for learning, for identifying the emergence of processes of argumentation for learning.